MATH 557 - ASSIGNMENT 1

To be handed in not later than 5pm, 24th January 2008.
Please hand in during lectures, to Burnside 1235, or to the Mathematics Office Burnside 1005

1 Find sufficient statistics for random samples of size n from the following distributions. Note that the sufficient statistics may be multidimensional, but must have dimension no greater than two.

(a) The Beta density with parameters α and β

$$f_{X|\alpha,\beta}(x|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \qquad 0 < x < 1$$

and zero otherwise, for $\alpha, \beta > 0$.

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(b) The pmf

$$f_{X|\theta}(x|\theta) = \frac{(\log\theta)^x}{\theta x!} \qquad x = 0, 1, 2, \ldots$$

and zero otherwise, for $\theta > 1$.

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(c) The Uniform density given by

$$f_{X|\theta}(x|\theta) = \frac{1}{\theta} \qquad \theta < x < 2\theta$$

and zero otherwise, for $\theta > 0$.

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2 Suppose that $X_1, \ldots, X_n$ are a random sample from an $Exponential(\lambda)$ distribution,

$$f_{X|\lambda}(x|\lambda) = \lambda e^{-\lambda x} \qquad x > 0$$

and zero otherwise, for $\lambda > 0$.

(a) Derive a minimal sufficient statistic for λ.

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(b) Suppose now that only the m smallest of the X_is are observed. Derive a sufficient statistic for λ based on this reduced sample of size m.

Hint: Construct the joint density of the first m order statistics $X_{(1)}, \ldots, X_{(m)}$ by noting that

$$X_{(m)} = x \qquad \Longleftrightarrow \qquad X_{(r)} > x, \text{ for all } r > m.$$

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3 Suppose that $X_1, \ldots, X_n$ are a random sample from a $Poisson(\theta)$ distribution. It can be shown that

$$T(\underset{\sim}{X}) = \sum_{i=1}^{n} X_i$$

is a sufficient statistic for θ.

Find the conditional mass function of $\underset{\sim}{X}$ given that $T(\underset{\sim}{X}) = t$.

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