MATH 556 - ASSIGNMENT 1

To be handed in not later than 5pm, 20th September 2007.
Please hand in during lectures, to Burnside 1235, or to the Mathematics Office Burnside 1005

1. Suppose that X is a discrete random variable with pmf f_X specified by

$$f_X(x) = \frac{k}{x(x+1)} \qquad x = 1, 2, 3, \ldots$$

and zero otherwise. Find k, and plot/sketch f_X and the corresponding cdf, F_X.

5 MARKS

2. The *hazard function*, h_X, for a discrete random variable X with support $\mathbb{X} = \{1, 2, 3, \ldots\}$ is defined by the formula

$$h_X(x) = \frac{f_X(x)}{1 - F_X(x-1)} \qquad x \in \mathbb{X}$$

where f_X and F_X are the pmf and cdf for X.

Give an interpretation of h_X as a conditional probability, show that $0 \le h_X(x) \le 1$, and find an f_X such that h_X is a constant for $x \in \mathbb{X}$.

5 MARKS

3. Suppose that F_X is the distribution function for random variable X, and

$$F_X(x) = \begin{cases} 0 & \text{if } x < -1 \\ 1 - \theta & \text{if } -1 \le x < 0 \\ 1 - \theta + \theta x/2 & \text{if } 0 \le x \le 2 \\ 1 & \text{if } x > 2 \end{cases}$$

for some parameter θ. Find

  (i) $\Pr[X = -1]$
 (ii) $\Pr[X = 0]$
(iii) $\Pr[X \ge 1]$

5 MARKS

4. Suppose that X is a continuous random variable with pdf, f_X, defined by

$$f_X(x) = \frac{2}{\pi} \qquad 0 < x < \pi/2$$

and zero otherwise. Prove that the function F defined for $0 < y < 1$ by

$$F(y) = \Pr[\sin(X) \le y]$$

specifies a cdf for a continuous random variable with support $(0, 1)$. Find the corresponding pdf.

5 MARKS