MATH 556 - EXERCISES 1

These exercises are not for assessment

1. For which values of the constant c do the following functions define valid probability mass functions for a discrete random variable X, taking values on range $\mathbb{X} = \{1, 2, 3, ...\}$:

$$\text{(a) } f_X(x) = c/2^x \qquad \text{(b) } f_X(x) = c/(x2^x)$$

$$\text{(c) } f_X(x) = c/(x^2) \qquad \text{(d) } f_X(x) = c2^x/x!$$

In each case, calculate (where possible) P$[X > 1]$ and P$[X \text{ is even}]$

2. n identical fair coins are tossed. Those that show Heads are tossed again, and the number of Heads obtained on the second set of tosses defines a discrete random variable X. Assuming that all tosses are independent, find the support $\mathbb{X}$ and probability mass function, f_X of X. *Hint: recall the Binomial distribution.*

3. Suppose that F_X is a cdf for random variable X. Let r be a positive integer. Decide whether each of the following functions is also a valid cdf:

(a) $F(x) = \{F_X(x)\}^r$ (b) $F(x) = 1 - \{1 - F_X(x)\}^r$

(c) $F(x) = F_X(x) + \{1 - F_X(x)\} \log\{1 - F_X(x)\}$ (d) $F(x) = \{F_X(x) - 1\}e + \exp\{1 - F_X(x)\}$

4. A continuous random variable X has pdf given by

$$f_X(x) = c(1-x)x^2 \quad 0 < x < 1$$

and zero otherwise. Find the value of c, the cdf F_X, and

$$\text{P}[X > 1/2].$$

5. A continuous random variable X has pdf given by

$$f_X(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 \leq x < 2 \end{cases}$$

and zero otherwise. Sketch f_X, and find the cdf F_X.

6. A continuous random variable X has cdf given by

$$F_X(x) = c(\alpha x^\beta - \beta x^\alpha) \quad 0 \leq x \leq 1$$

for constants $1 \leq \beta < \alpha$, with the usual behaviour for a cdf elsewhere. Find the value of constant c, and evaluate the rth moment of X.

7. A continuous random variable X has cdf given by

$$F_X(x) = \frac{2\beta x}{\beta^2 + x^2} \quad 0 \leq x \leq \beta$$

for constant $\beta > 0$, with the usual behaviour for a cdf elsewhere. Find the pdf of X, and show that the expectation of X is

$$\beta(1 - \log 2)$$