MATH 556 - ASSIGNMENT 4

To be handed in not later than 5pm, 30th November 2006.
Please hand in during lectures, to Burnside 1235, or to the Mathematics Office Burnside 1005

1 Suppose that X has expectation zero, and finite variance σ^2. Prove that, for $t > 0$,

$$P[X \geq t] \leq \frac{\sigma^2}{\sigma^2 + t^2}$$

4 MARKS

2 Suppose that $X_1, \ldots, X_n$ are a random sample from a Cauchy distribution, and let

$$Y_n = \max\{X_1, \ldots, X_n\}.$$

Find the limiting distribution (if it exists) of

(a) Y_n
(b) $T_n = \pi Y_n / n$.

Note, for any real x

$$\arctan(x) = x + o(x)$$

where $o(x)$ is a function such that

$$\lim_{x \longrightarrow 0} \frac{o(x)}{x} = 0.$$

that is, approximately, for small x

$$\arctan(x) \simeq x \ .$$

8 MARKS

3 Suppose that $X_1, X_2, \ldots, X_n, \ldots$ form a sequence of random variables with pdfs given, for $n \geq 1$, by

$$f_{X_n}(x) = \frac{1}{\pi} \frac{n}{1 + n^2 x^2} \qquad x \in \mathbb{R}.$$

Does X_n converge to zero

(a) in rth mean, for some r ?
(b) in probability ?

as $n \longrightarrow \infty$. Justify your answers.

5 MARKS

4 Prove that $X_n \xrightarrow{p} 0$ if and only if

$$E\left[\frac{|X_n|}{1 + |X_n|}\right] \longrightarrow 0 \qquad \text{as } n \longrightarrow \infty. \tag{1}$$

Method: First assume $X_n \xrightarrow{p} 0$ and prove that equation (1) holds, then prove the converse. Use the Chebychev Lemma/Markov's Inequality. Note that

$$x > \epsilon \quad \Longrightarrow \quad \frac{x}{1+x} > \frac{\epsilon}{1+\epsilon}$$

for $x, \epsilon > 0$.

8 MARKS