MATH 556 - ASSIGNMENT 3

To be handed in not later than 5pm, 16th November 2006.
Please hand in during lectures, to Burnside 1235, or to the Mathematics Office Burnside 1005

1 Consider the three-level hierarchical model:

LEVEL 3 :	$\lambda > 0, r \in \{1, 2, \ldots\}$	Fixed parameters
LEVEL 2 :	$N \sim Poisson(\lambda)$	
LEVEL 1 :	$X \vert N = n \sim Gamma(n + r/2, 1/2)$	

Find

(a) The expectation of X, $E_{f_X}[X]$,

3 MARKS

(b) The mgf of X, $M_X(t)$.

6 MARKS

2 Suppose that $X_1, \ldots, X_r$ are independent random variables such that, for each i, $X_i \sim N(\mu_i, 1)$, for fixed constants $\mu_1, \ldots, \mu_r$.

(a) Find the mgf of random variable Y defined by

$$Y = \sum_{i=1}^{r} X_i^2.$$

6 MARKS

(b) Find the skewness of Y, ς, where

$$\varsigma = \frac{E_{f_Y}[(Y - \mu)^3]}{\sigma^3}$$

where μ and σ^2 are the expectation and variance of f_Y.

6 MARKS

3 In a branching process model, the total number of individuals in successive generations are random variables $S_0, S_1, S_2, \ldots$. Suppose that, in the passage from generation i to generation $i + 1$, each of the s_i individuals observed in generation i gives rise to N_{ij} offspring for $j = 1, \ldots, s_i$ according to a pmf with corresponding pgf G_N.

In addition to the production of offspring, suppose that at each generation, immigration into the population is allowed, and that at generation i, K_i immigrants enter the population to go forward to the $i + 1$st generation, so that

$$S_{i+1} = \sum_{j=1}^{s_i} N_{ij} + K_i$$

where $K_0, K_1, K_2, \ldots$ are iid random variables, with pgf G_K, that are independent of all N_{ij}.

Find the pgf of S_{i+1} in terms of the pgf of random variable S_i and G_K.

4 MARKS