# MATH 204 - SOLUTIONS 1

The data yield the following statistics:

| Treatment | | 0 | 2 | 4 |
|---|---|---|---|---|
| Sample Size | $n_i$ | 4 | 5 | 5 |
| Sample Mean | $\overline{x}_i$ | 26.75 | 33.60 | 38.20 |
| Sample Variance | $s_i^2$ | 28.92 | 20.30 | 22.70 |

1. For the three two sample $t$-tests:

   (a) Groups 0 vs 2:

$$\begin{aligned} s_P^2 &= \frac{1}{n_1+n_2-2}\left[(n_1-1)s_1^2+(n_2-1)s_2^2\right] = \frac{1}{4+5-2}\left[3\times 28.92+4\times 20.30\right] = 23.99 \\ t &= \frac{\overline{x}_1-\overline{x}_2}{s_P\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{26.75-33.60}{4.90\sqrt{\frac{1}{4}+\frac{1}{5}}} = -2.085 \end{aligned}$$

   We compare this with the critical values of the Student-t$(n_1+n_2-2) \equiv$ Student-t$(7)$ distribution; we have from McClave and Sincich (page 896) that if $\alpha = 0.05$ then

$$C_R = \pm 2.365$$

   - we look up the 0.025 tail quantile in column 3. Hence we **do not reject** the hypothesis of equal means at $\alpha = 0.05$. Using software we find that the $p$-value is 0.076.

   (b) Groups 0 vs 4:

$$\begin{aligned} s_P^2 &= \frac{1}{n_1+n_3-2}\left[(n_1-1)s_1^2+(n_3-1)s_3^2\right] = \frac{1}{4+5-2}\left[3\times 28.92+4\times 22.70\right] = 25.36 \\ t &= \frac{\overline{x}_1-\overline{x}_3}{s_P\sqrt{\frac{1}{n_1}+\frac{1}{n_3}}} = \frac{26.75-38.20}{5.03\sqrt{\frac{1}{4}+\frac{1}{5}}} = -3.389 \end{aligned}$$

   We compare this with the critical values of the Student-t$(n_1+n_3-2) \equiv$ Student-t$(7)$ distribution; if $\alpha = 0.05$ then again $C_R = \pm 2.365$. Hence we **reject** the hypothesis of equal means at $\alpha = 0.05$.

   Using software we find that the $p$-value is 0.011, so if instead we were to choose $\alpha = 0.01$ ($C_R = \pm 3.499$) then we would not reject the null hypothesis of equal means.

   (c) Groups 2 vs 4:

$$\begin{aligned} s_P^2 &= \frac{1}{n_2+n_3-2}\left[(n_2-1)s_2^2+(n_3-1)s_3^2\right] = \frac{1}{5+5-2}\left[4\times 20.30+4\times 22.70\right] = 21.50 \\ t &= \frac{\overline{x}_1-\overline{x}_2}{s_P\sqrt{\frac{1}{n_2}+\frac{1}{n_3}}} = \frac{33.60-38.20}{4.64\sqrt{\frac{1}{5}+\frac{1}{5}}} = -1.569 \end{aligned}$$

   We compare this with the critical values of the Student-t$(n_2+n_3-2) \equiv$ Student-t$(8)$ distribution; we have from McClave and Sincich (page 896) that if $\alpha = 0.05$ then

$$C_R = \pm 2.306$$

   Hence we **do not reject** the hypothesis of equal means at $\alpha = 0.05$. Using software we find that the $p$-value is 0.155.

2. By direct calculation

$$\begin{aligned} \text{SST} &= 292.1071 \\ \text{SSE} &= 258.7500 \\ \text{SS} &= 550.8571 \end{aligned}$$

so that

$$\text{MST} = \frac{\text{SST}}{k-1} = \frac{292.1071}{2} = 146.054$$

$$\text{MSE} = \frac{\text{SSE}}{n-k} = \frac{258.7500}{11} = 23.523$$

and

$$F = \frac{\text{MST}}{\text{MSE}} = \ = 6.209.$$

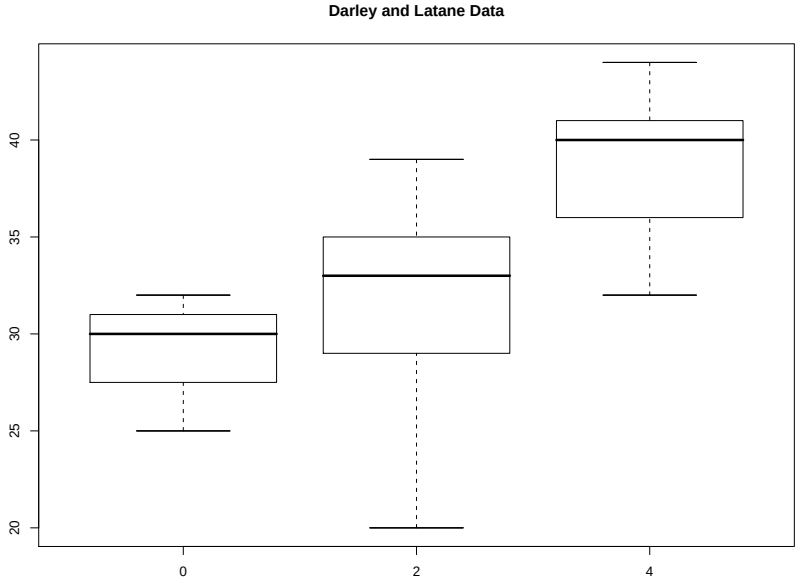
We compare this with the $1-\alpha = 0.95$ point of the Fisher-F$(k-1, n-k)$ = Fisher-F$(2, 11)$ distribution. From tables (*McClave and Sincich*, p. 901)

$$F_\alpha(2, 11) = 3.98$$

and hence we

**REJECT** $H_0$

at the $\alpha = 0.05$ significance level.




From the boxplot, we can see that there is no real evidence to think that the assumptions behind the ANOVA F-test are not valid here.

Two sample t-tests

## Group 0 vs Group 2

**Group Statistics**

| | Number of People in Room | N | Mean | Std. Deviation | Std. Error Mean |
|---|---|---|---|---|---|
| Response Time (s) | ZERO | 4 | 26.75 | 5.377 | 2.689 |
| | TWO | 5 | 33.60 | 4.506 | 2.015 |

**Independent Samples Test**

| | | Levene's Test for Equality of Variances | | t-test for Equality of Means | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | 95% Confidence Interval of the Difference | |
| | | F | Sig. | t | df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | Lower | Upper |
| Response Time (s) | Equal variances assumed | .246 | .635 | -2.085 | 7 | .076 | -6.850 | 3.286 | -14.620 | .920 |
| | Equal variances not assumed | | | -2.039 | 5.916 | .088 | -6.850 | 3.360 | -15.100 | 1.400 |

## Group 0 vs Group 4

**Group Statistics**

| | Number of People in Room | N | Mean | Std. Deviation | Std. Error Mean |
|---|---|---|---|---|---|
| Response Time (s) | ZERO | 4 | 26.75 | 5.377 | 2.689 |
| | FOUR | 5 | 38.20 | 4.764 | 2.131 |

**Independent Samples Test**

| | | Levene's Test for Equality of Variances | | t-test for Equality of Means | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | 95% Confidence Interval of the Difference | |
| | | F | Sig. | t | df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | Lower | Upper |
| Response Time (s) | Equal variances assumed | .108 | .752 | -3.389 | 7 | .012 | -11.450 | 3.378 | -19.439 | -3.461 |
| | Equal variances not assumed | | | -3.338 | 6.136 | .015 | -11.450 | 3.431 | -19.799 | -3.101 |

## Group 2 vs Group 4

**Group Statistics**

| | Number of People in Room | N | Mean | Std. Deviation | Std. Error Mean |
|---|---|---|---|---|---|
| Response Time (s) | TWO | 5 | 33.60 | 4.506 | 2.015 |
| | FOUR | 5 | 38.20 | 4.764 | 2.131 |

**Independent Samples Test**

| | | Levene's Test for Equality of Variances | | t-test for Equality of Means | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | 95% Confidence Interval of the Difference | |
| | | F | Sig. | t | df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | Lower | Upper |
| Response Time (s) | Equal variances assumed | .029 | .868 | -1.569 | 8 | .155 | -4.600 | 2.933 | -11.363 | 2.163 |
| | Equal variances not assumed | | | -1.569 | 7.975 | .155 | -4.600 | 2.933 | -11.366 | 2.166 |

One-Way ANOVA and ANOVA F-test

# Explore

## Number of People in Room

**Case Processing Summary**

| | Number of People in Room | Cases | | | | | |
|---|---|---|---|---|---|---|---|
| | | Valid | | Missing | | Total | |
| | | N | Percent | N | Percent | N | Percent |
| Response Time (s) | ZERO | 4 | 100.0% | 0 | .0% | 4 | 100.0% |
| | TWO | 5 | 100.0% | 0 | .0% | 5 | 100.0% |
| | FOUR | 5 | 100.0% | 0 | .0% | 5 | 100.0% |

# Oneway

**Descriptives**

Response Time (s)

| | N | Mean | Std. Deviation | Std. Error | 95% Confidence Interval for Mean | | Minimum | Maximum |
|---|---|---|---|---|---|---|---|---|
| | | | | | Lower Bound | Upper Bound | | |
| ZERO | 4 | 26.75 | 5.377 | 2.689 | 18.19 | 35.31 | 20 | 32 |
| TWO | 5 | 33.60 | 4.506 | 2.015 | 28.01 | 39.19 | 29 | 40 |
| FOUR | 5 | 38.20 | 4.764 | 2.131 | 32.28 | 44.12 | 32 | 44 |
| Total | 14 | 33.29 | 6.510 | 1.740 | 29.53 | 37.04 | 20 | 44 |

**Test of Homogeneity of Variances**

Response Time (s)

| Levene Statistic | df1 | df2 | Sig. |
|---|---|---|---|
| .123 | 2 | 11 | .885 |

**ANOVA**

Response Time (s)

| | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|
| Between Groups | 292.107 | 2 | 146.054 | 6.209 | .016 |
| Within Groups | 258.750 | 11 | 23.523 | | |
| Total | 550.857 | 13 | | | |