Note: For two factors A and B, the **main effects plus interaction** model can be written

$$\mathrm{A} + \mathrm{B} + \mathrm{A.B}$$

whereas the **main effects only** can be written

$$\mathrm{A} + \mathrm{B}$$

The models

$$\mathrm{A} + \mathrm{A.B} \qquad\qquad \mathrm{B} + \mathrm{A.B}$$

do not make sense.

For a two factor design, the only models that should be considered and or reported are

MODEL	FACTOR	INTERACTION
NULL	NONE	NONE
A	A	NONE
B	B	NONE
A+B	A,B	NONE
A+B+A.B	A,B	YES

that is, if the interaction is significant, the only model you should report is

$$A + B + A.B$$

Note: ANOVA analysis for the RBD and FD (both with replication) are identical. The only difference lies in the interpretation of the factors

- RBD: one blocking, one treatment factor
- FD: two treatment factors

"Blocking" factors are known or strongly believed to have a significant effect on the response.

Estimating Effect Size

In multifactor designs, parameter estimation can be carried out in different parameterizations

For the CRD (one-way layout):

- Natural parameters: $\mu_1, \ldots, \mu_k$
- Contrast parameters: $\beta, \beta_0, \ldots, \beta_{k-1}$ where

$$\beta = \mu_k \qquad \beta_i = \mu_i - \mu_k, \quad i = 1, \ldots, k-1$$

that is, differences from baseline.

For the two-factor designs (RBD/FD): In the two-way layout, with cells (i, j), $i = 1, \ldots, a$, $j = 1, \ldots, b$. The cell means are m_{ij}, where

$$m_{ij} = \mu_{i.} + \mu_{.j} + \mu_{ij}$$

where $\mu_{i.}$ gives the Factor A contribution, $\mu_{.j}$ gives the Factor B contribution, and μ_{ij} gives the interaction.

The parameterization used by SPSS is the contrast parameterization is

$$\begin{aligned} m_{ij} &= \beta_0 && i = a, j = b \\ &= \beta_0 + \beta_i^{(A)} && i = 1, \ldots, a-1, j = b \\ &= \beta_0 + \beta_j^{(B)} && i = a, j = 1, \ldots, b-1 \\ &= \beta_0 + \beta_i^{(A)} + \beta_j^{(B)} + \gamma_{ij}^{(AB)} \\ & && i = 1, \ldots, a-1 \\ & && j = 1, \ldots, b-1 \end{aligned}$$

where

$\beta_i^{(A)}$: contrasts for factor A
$\beta_j^{(B)}$: contrasts for factor B
$\gamma_{ij}^{(AB)}$: interaction

SPSS takes the *a*th level of factor A and the *b*th level of factor B as the baseline, and looks at differences compared to this baseline.

The *ab* parameters are

β_0	1
$\beta_1^{(A)}, \ldots, \beta_{a-1}^{(A)}$	$(a-1)$
$\beta_1^{(B)}, \ldots, \beta_{b-1}^{(B)}$	$(b-1)$
$\gamma_{ij}^{(AB)}, i = 1, \ldots, a-1, j = 1, \ldots, b-1$	$(a-1)(b-1)$
Total	ab

For example: $a = 3, b = 4$.

		Factor B			
		1	2	3	4
Factor A	1	①	②	③	$\beta_0 + \beta_1^{(A)}$
	2	④	⑤	⑥	$\beta_0 + \beta_2^{(A)}$
	3	$\beta_0 + \beta_1^{(B)}$	$\beta_0 + \beta_2^{(B)}$	$\beta_0 + \beta_3^{(B)}$	β_0

where

① $= \beta_0 + \beta_1^{(A)} + \beta_1^{(B)} + \gamma_{11}^{(AB)}$

⑥ $= \beta_0 + \beta_2^{(A)} + \beta_3^{(B)} + \gamma_{23}^{(AB)}$

and so on.

Estimation is still straightforward:

PARAMETER	ESTIMATE
β_0	$\overline{x}_{ab}$
$\beta_i^{(A)}$	$\overline{x}_{i.} - \overline{x}_{ab}$
$\beta_j^{(A)}$	$\overline{x}_{.j} - \overline{x}_{ab}$
$\gamma_{ij}^{(AB)}$	$\overline{x}_{ij} - \overline{x}_{i.} - \overline{x}_{.j} + \overline{x}_{ab}$

for $i = 1, \ldots, a$, $j = 1, \ldots, b$.

Other parameterizations can be used.

Final Note on ANOVA

We have studied the simplest design scenarios: extension to

- *incomplete*
- *unbalanced*
- *nested*
- *random effect*

designs are possible.

Furthermore SPSS has greater functionality: for example, it has the capability to carry out ANOVA-like analyses even for the case of non-equal variances (when Levene's test rejects the hypothesis of equal variances).