Comment: The "sum of squares" decompositions

$$
\begin{aligned}
CRD : SS &= [SST] + SSE \\
RBD : SS &= [SST + SSB] + SSE
\end{aligned}
$$

are both of the form

TOTAL VARIATION	=	SYSTEMATIC VARIATION	+	RANDOM VARIATION

"SYSTEMATIC" { For the CRD: SST ; For the RBD: $SST + SSB$ }

"RANDOM" For both: SSE

We have studied the

Randomized **Complete** Block Design

where each block/treatment combination has one experimental unit.

An incomplete design could also be considered, where some block/treatment combinations are omitted. However, this design does not lead to straightforward ANOVA analysis.

1.5 Factorial Experiments

Designs studied so far:

- CRD - one factor
- RBD - one factor, plus one blocking variable, so two factors in total, where one (the blocking variable) is a known source of systematic variation.

However, in the RBD, we must assume that the treatments behave in a similar way across blocks.

Let i index treatments $(1 \leq i \leq k)$ and consider block j, and two treatment (factor levels) i_1 and i_2.

In an RBD, we assume that

$$E[X_{i_1 j} - X_{i_2 j}] = \mu_{i_1} - \mu_{i_2}$$

which does **NOT** depend on j.

That is, the expected difference in response due to the two treatments does not depend on the block.

But perhaps the difference **does** depend on block; perhaps we have **INTERACTION**.

In the current RBD, we do not have enough data to look for this. We now seek to extend the RBD to allow for tests for interaction; we do this by using **replication**.

RBD with Balanced Replication

Suppose we have r observations per block/treatment combination (termed *replicates*), so that we have $n - bkr$ experimental units in total.

Balanced designs have equal numbers of replicates in each block/treatment combination.

In this design, all the quantities

$$SST, SSB, SSE, SS$$
$$MST, MSB, MSE$$

can be defined, and an ANOVA F-test can be carried out - the only difference is that $n = bkr$.

- Sum of Squares for Treatments (SST)

$$SST = \sum_{i=1}^{k} br(\overline{x}_i - \overline{x})^2$$

- Sum of Squares for Blocks (SSB)

$$SSB = \sum_{j=1}^{b} kr(\overline{x_j^{(B)}} - \overline{x})^2$$

- Overall Sum of Squares (SS)

$$SS = \sum_{i=1}^{k} \sum_{j=1}^{b} \sum_{t=1}^{r} (x_{ijt} - \overline{x})^2$$

and $SSE = SS - SST - SSB$

Third index t indexes the replicates.

The RBD with replication does allow the investigation of interaction. The new test is based on the decomposition

$$SS = SST + SSB + SSI + SSE$$

where SSI is the sum of squares for Interaction.

We have SST, SSB and SS as before, and

$$SSI = \sum_{i=1}^{k} \sum_{j=1}^{b} r(\overline{x}_{ij} - \overline{x}_i - \overline{x_j^{(\mathrm{B})}} + \overline{x})^2$$

where

$$\overline{x}_{ij} = \frac{1}{r} \sum_{t=1}^{r} x_{ijt} \qquad i = 1, \ldots, k, \; j = 1, \ldots, b$$

is the sample mean for replicates in (i, j)th treatment/block combination.