

RANDOMIZED BLOCK DESIGNS AND THE ANOVA F-TEST

Consider a **randomized block design** (RBD) with k treatments and b blocks. Assume that each block has k experimental units, and that one unit is assigned to each treatment. Let x_{ij} be the measured response for the experimental unit from block j in treatment i and

- sample mean for **treatment** i

$$\bar{x}_i = \frac{1}{b} \sum_{j=1}^b x_{ij} \quad i = 1, \dots, k$$

- sample mean for **block** j

$$\overline{x_j^{(B)}} = \frac{1}{k} \sum_{i=1}^k x_{ij} \quad j = 1, \dots, b$$

- overall sample mean

$$\bar{x} = \frac{1}{n} \sum_{i=1}^k \sum_{j=1}^b x_{ij}$$

- Sum of Squares for **Treatments** (SST)

$$SST = \sum_{i=1}^k b(\bar{x}_i - \bar{x})^2$$

- Sum of Squares for **Blocks** (SSB)

$$SSB = \sum_{j=1}^b k(\overline{x_j^{(B)}} - \bar{x})^2$$

- Overall Sum of Squares (SS)

$$SS = \sum_{i=1}^k \sum_{j=1}^b (x_{ij} - \bar{x})^2$$

The following decomposition holds

$$SS = SST + SSB + SSE \quad \therefore \quad SSE = SS - SST - SSB$$

For testing

$$H_0 : \mu_1 = \dots = \mu_k$$

$$H_a : \text{At least two treatment means different}$$

in an RBD, the test statistic is

$$F = \frac{MST}{MSE}$$

where

$$MST = \frac{SST}{k-1} \quad MSE = \frac{SSE}{n-b-k+1}$$

If H_0 is **true**, then $F \sim \text{Fisher-F}(k-1, n-b-k+1)$, and the rejection region for the test with significance level α is

$$F > F_\alpha(k-1, n-b-k+1)$$

where $F_\alpha(\nu_1, \nu_2)$ is the $1 - \alpha$ percentage point of the Fisher-F distribution with ν_1 and ν_2 degrees of freedom (see pages 899-905 of McClave and Sincich 10th Ed)

EXAMPLE

Data: Measurements were made on the amount of sulphur (in parts per million) in soil samples using four different solvents. The soil samples were collected from five different geographical locations in Florida, USA, and represented different soil types.

The **response variable** sulphur level. The single **factor** is the *solvent* and there are $k = 4$ **factor levels**:

1. Calcium Chloride (CaCl_2)
2. Ammonium Acetate (NH_4OAc)
3. Mono-Calcium Phosphate ($\text{Ca}(\text{H}_2\text{P O}_4)_3$)
4. Water (H_2O)

The *soil* types determine the $b = 5$ **blocks**

1. Troup, Jackson Co. (*Paleudults* soil)
2. Lakeland, Walton Co. (*Quartzipsamments* soil)
3. Leon, Duval Co. (*Haplaquads* soil)
4. Chipley, Jackson Co. (*Quartzipsamments* soil)
5. Norfolk, Alachua Co. (*Paleudults* soil)

The data observed in the study were as follows:

Treatment	Block				
	Troup	Lakeland	Leon	Chipley	Norfolk
CaCl_2	5.07	3.31	2.54	2.34	4.71
NH_4OAc	4.43	2.74	2.09	2.07	5.29
$\text{Ca}(\text{H}_2\text{P O}_4)_3$	7.09	2.32	1.09	4.38	5.70
H_2O	4.48	2.35	2.70	3.85	4.98

Using SPSS, the following ANOVA table was obtained; see the related SPSS screens at

www.math.mcgill.ca/~dstephens/204/Handouts/Math204-SPSS-RBDANOVA-Screens.pdf

Tests of Between-Subjects Effects

Dependent Variable: Sulphur content (ppm)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	35.586(a)	7	5.084	6.327	.003
Intercept	270.333	1	270.333	336.460	.000
solvent	1.621	3	.540	.673	.585
soil	33.965	4	8.491	10.568	.001
Error	9.642	12	.803		
Total	315.561	20			
Corrected Total	45.228	19			

a. R Squared = .787 (Adjusted R Squared = .662)

This table contains a much information not needed for the ANOVA F-test; the rows headed

- Corrected Model (row 1)
- Intercept (row 2)
- Total (row 6)

can be ignored. The remaining rows are the standard ANOVA table for the randomized block design. As expected, there is a significant difference between **blocks** (row 4, $F = 10.568$, p -value=0.001), but **no significant difference** between **treatments** (row 3, $F = 0.673$, p -value=0.585).