Faculty of Science
FINAL EXAMINATION
MATH 204B
Principles of Statistics 2

Examiner: Professor K.J. Worsley
Associate Examiner: Professor R. Steele

Date: Friday, April 28, 2005
Time: 14:00 - 17:00 hours

INSTRUCTIONS: Answer all questions. An 8.5×11 inch piece of paper, written on both sides, may be brought into the exam. Calculators are permitted. Computer printout, figures and tables are provided at the end of the exam.
Use of a regular and or translation dictionary is permitted.
Each part of each question is worth approximately equal marks.

This exam comprises this cover, 3 pages of questions, 8 pages of computer printout, and 3 pages of tables (15 pages in all).

1. The article "Responsiveness of Food Sales to Shelf Space Changes in Supermarkets" (*J. Mktg. Res.* (1964): 63-67) described an experiment to assess the effect of allotted shelf space sales of "Tang". Six stores were used in the experiment, and six different shelf-space allotments of 6, 9, 12, 15, 18 and 21 feet were tried for 1 week. The author speculated that changes in shelf space would affect sales. Data on the number of containers of Tang sold is given in the table below.

	Shelf space (feet)					
Store	6	9	12	15	18	21
1	30	35	25	27	38	31
2	47	59	43	62	65	48
3	47	55	48	54	36	54
4	29	19	41	27	33	39
5	17	11	25	23	24	26
6	22	9	19	18	25	22

Choose appropriate pairs of models, or T-statistics, to answer the following.

(a) Is there any effect of shelf space (as a factor) on sales, allowing for different sales in different stores? Test this.

(b) The table on the printout labelled "Multiple Comparisons" tests for differences between all pairs of shelf space, allowing for different sales in different stores. Are there any differences?

(c) Is there any effect of shelf space (as a linear covariate) on sales, allowing for different sales in different stores? Test this.

(d) Explain carefully why there is a discrepancy in your answers to (a), (b) and (c).

(e) Does the effect of shelf space (as a linear covariate) differ from one store to another? Test this.

(f) Store 1 is thinking of devoting 10 feet of shelf space to Tang. Use what you think is the most appropriate model to predict the weekly sales of Tang. Give an approximate standard deviation for your prediction.

(g) Tricky question: Why can we *not* test that the effect shelf space (as a factor) differs from one store to another?

2. The so-called fog index is intended to measure how difficult it is to read a piece of writing. It is calculated as $0.4\times$ (average number of words per sentence) + (percentage of words with three or more syllables). Shruptine and McVicker (1981) made a study of the readability of magazine advertisements in which they obtained fog indices from a random sample of six advertisements from *Scientific American* and *Newsweek*, obtaining the results in the table below:

	Fog indices					
Scientific American	15.75	11.55	8.20	9.23	11.16	9.92
Newsweek	9.66	7.67	10.21	5.12	3.12	4.88

Carry out a suitable non-parametric test to see if there is any difference in fog indices between *Scientific American* and *Newsweek*. **There is no printout for this question.**

3. The table below gives the number of patients with malignant melanoma cross classified by Type (the type of abnormal cells making up the melanoma) and Site (where on the body the cancer appeared).

	Site		
Type	Extremities	Head and Neck	Trunk
Hutchinson's melanomic freckle	10	22	2
Intermediate	28	11	17
Nodular	73	19	33
Superficial spreading melanoma	115	16	54

(a) Estimate the expected number of cases of superficial spreading melanoma in the trunk, assuming that Type and Site are independent (this cell is empty on the printout).

(b) Is there any evidence that the proportion of types of melanoma is different at different sites? Test this.

4. The data for this question originated from a car's journey log kept by a brother of statistics professor Jack Robinson at Waterloo over a 12 month period. Each time the car was filled with petrol, five variables were measured:

- Dist, the distance run since the last fill-up, in kilometres.
- KmPerL, the fuel efficiency, in kilometres per liter.
- Days, the number of days siince the last fill-up.
- Month, the month of the year (January=1, ..., December=12).
- Temp, the average monthly temperature in Waterloo.

There were 86 measurements in total, and the first 20 appear on the printout. Our aim is to predict the fuel efficiency, KmPerL, as a function of the other variables. Choose appropriate pairs of models, or T-statistics, to test the following.

(a) Ignoring Temp for the moment, a model was fitted with Month as a factor and Dist and Days as covariates. Is there any effect of evidence of an effect of

i. Dist?

ii. Days?

iii. Month?

allowing for an effect of the other two.

(b) Month was replaced by Temp in the model. Is there an effect of Temp, allowing for Dist and Days?

(c) Month was added back into the model, together with Dist, Days and Temp. Does the mean monthly temperature explain all the effect of month, or putting it another way, can the effect of Month be explained solely by a linear effect of Temp?

(d) The graph is a plot of residuals from the model fitted in (b), against the fitted values. Do you think the assumptions of normality and equal variance are satisfied?

(e) What proportion of the variance of KmPerL is explained by the model in (b)?

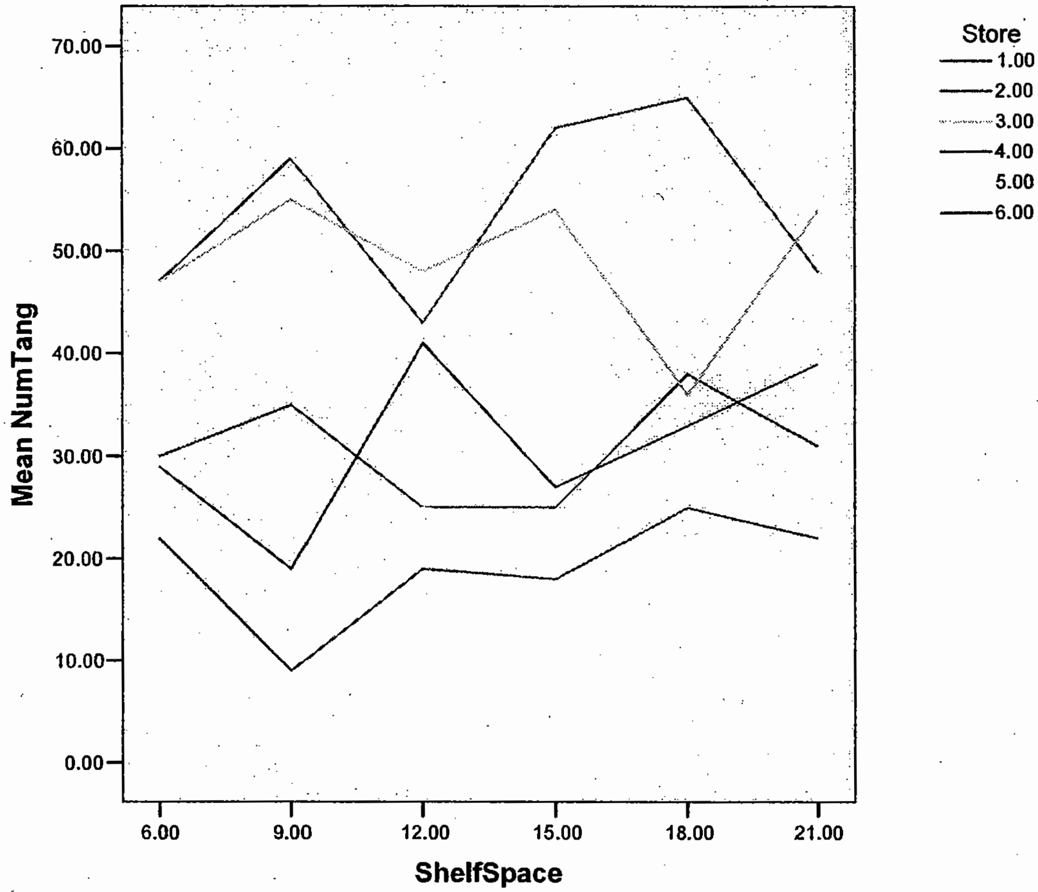
(f) Use a suitable model to predict the fuel efficiency of a long trip of 300 km with 0 days between fill-ups in April (Month=4) when the average temperature is 5.80 degrees. Give an approximate standard deviation for your prediction.

(g) Tricky question: In the last model fitted, explain exactly why [Month=11.00] has a coefficient of 0 and is redundant.

Question 1. Tang

Case Summaries

	NumTang	Store	ShelfSpace
1	30.00	1.00	6.00
2	47.00	2.00	6.00
3	47.00	3.00	6.00
4	29.00	4.00	6.00
5	17.00	5.00	6.00
6	22.00	6.00	6.00
7	35.00	1.00	9.00
8	59.00	2.00	9.00
9	55.00	3.00	9.00
10	19.00	4.00	9.00
11	11.00	5.00	9.00
12	9.00	6.00	9.00
13	25.00	1.00	12.00
14	43.00	2.00	12.00
15	48.00	3.00	12.00
16	41.00	4.00	12.00
17	25.00	5.00	12.00
18	19.00	6.00	12.00
19	25.00	1.00	15.00
20	62.00	2.00	15.00
21	54.00	3.00	15.00
22	27.00	4.00	15.00
23	23.00	5.00	15.00
24	18.00	6.00	15.00
25	38.00	1.00	18.00
26	65.00	2.00	18.00
27	36.00	3.00	18.00
28	33.00	4.00	18.00
29	24.00	5.00	18.00
30	25.00	6.00	18.00
31	31.00	1.00	21.00
32	48.00	2.00	21.00
33	54.00	3.00	21.00
34	39.00	4.00	21.00
35	26.00	5.00	21.00
36	22.00	6.00	21.00
Total N	36	36	36



Error SS = 7661.639, df = 35

Dependent Variable: NumTang

Parameter	B	Std. Error	t	Sig.
Intercept	34.194	2.466	13.867	.000

Error SS = 1469.50, df = 30

Dependent Variable: NumTang

Parameter	B	Std. Error	t	Sig.
Intercept	19.167	2.857	6.708	.000
[Store=1.00]	11.500	4.041	2.846	.008
[Store=2.00]	34.833	4.041	8.620	.000
[Store=3.00]	29.833	4.041	7.383	.000
[Store=4.00]	12.167	4.041	3.011	.005
[Store=5.00]	1.833	4.041	.454	.653
[Store=6.00]	0(a)	.	.	.

a This parameter is set to zero because it is redundant.

Error SS = 1307.694, df = 25

Dependent Variable: NumTang

Parameter	B	Std. Error	t	Sig.
Intercept	21.639	3.998	5.413	.000
[Store=1.00]	11.500	4.176	2.754	.011
[Store=2.00]	34.833	4.176	8.342	.000
[Store=3.00]	29.833	4.176	7.145	.000
[Store=4.00]	12.167	4.176	2.914	.007
[Store=5.00]	1.833	4.176	.439	.664
[Store=6.00]	0(a)	.	.	.
[ShelfSpace=6.00]	-4.667	4.176	-1.118	.274
[ShelfSpace=9.00]	-5.333	4.176	-1.277	.213
[ShelfSpace=12.00]	-3.167	4.176	-.758	.455
[ShelfSpace=15.00]	-1.833	4.176	-.439	.664
[ShelfSpace=18.00]	.167	4.176	.040	.968
[ShelfSpace=21.00]	0(a)	.	.	.

a This parameter is set to zero because it is redundant.

Multiple Comparisons

Dependent Variable: NumTang
LSD

(I) ShelfSpace	(J) ShelfSpace	Mean Difference (I-J)	Std. Error	Sig.
6.00	9.00	.6667	4.17563	.874
	12.00	-1.5000	4.17563	.722
	15.00	-2.8333	4.17563	.504
	18.00	-4.8333	4.17563	.258
	21.00	-4.6667	4.17563	.274
9.00	6.00	-.6667	4.17563	.874
	12.00	-2.1667	4.17563	.608
	15.00	-3.5000	4.17563	.410
	18.00	-5.5000	4.17563	.200
	21.00	-5.3333	4.17563	.213
12.00	6.00	1.5000	4.17563	.722
	9.00	2.1667	4.17563	.608
	15.00	-1.3333	4.17563	.752
	18.00	-3.3333	4.17563	.432
	21.00	-3.1667	4.17563	.455
15.00	6.00	2.8333	4.17563	.504
	9.00	3.5000	4.17563	.410
	12.00	1.3333	4.17563	.752
	18.00	-2.0000	4.17563	.636
	21.00	-1.8333	4.17563	.664
18.00	6.00	4.8333	4.17563	.258
	9.00	5.5000	4.17563	.200
	12.00	3.3333	4.17563	.432

	15.00	2.0000	4.17563	.636
	21.00	.1667	4.17563	.968
21.00	6.00	4.6667	4.17563	.274
	9.00	5.3333	4.17563	.213
	12.00	3.1667	4.17563	.455
	15.00	1.8333	4.17563	.664
	18.00	-.1667	4.17563	.968

Based on observed means.

Error SS = 7516.379, df = 34

Dependent Variable: NumTang

Parameter	B	Std. Error	t	Sig.
Intercept	28.902	6.984	4.138	.000
ShelfSpace	.392	.484	.811	.423

Error SS = 1324.24, df = 29

Dependent Variable: NumTang

Parameter	B	Std. Error	t	Sig.
Intercept	13.874	4.052	3.424	.002
[Store=1.00]	11.500	3.901	2.948	.006
[Store=2.00]	34.833	3.901	8.928	.000
[Store=3.00]	29.833	3.901	7.647	.000
[Store=4.00]	12.167	3.901	3.119	.004
[Store=5.00]	1.833	3.901	.470	.642
[Store=6.00]	0(a)	.	.	.
ShelfSpace	.392	.220	1.784	.085

a This parameter is set to zero because it is redundant.

Error SS = 1223.314, df = 24

Dependent Variable: NumTang

Parameter	B	Std. Error	t	Sig.
Intercept	13.124	8.214	1.598	.123
[Store=1.00]	15.743	11.617	1.355	.188
[Store=2.00]	35.476	11.617	3.054	.005
[Store=3.00]	37.933	11.617	3.265	.003
[Store=4.00]	8.181	11.617	.704	.488
[Store=5.00]	-2.667	11.617	-.230	.820
[Store=6.00]	0(a)	.	.	.
ShelfSpace	.448	.569	.787	.439
[Store=1.00] * ShelfSpace	-.314	.805	-.391	.700
[Store=2.00] * ShelfSpace	-.048	.805	-.059	.953
[Store=3.00] * ShelfSpace	-.600	.805	-.746	.463
[Store=4.00] * ShelfSpace	.295	.805	.367	.717
[Store=5.00] * ShelfSpace	.333	.805	.414	.682
[Store=6.00] * ShelfSpace	0(a)	.	.	.

a This parameter is set to zero because it is redundant.

Question 2. Melanoma

Type * Site Crosstabulation

			Site			Total
			Extremities	Head and Neck	Trunk	
Type	Hutchinson's	Count	10	22	2	34
		Expected Count	19.2	5.8	9.0	34.0
	Intermediate	Count	28	11	17	56
		Expected Count	31.6	9.5	14.8	56.0
	Nodular	Count	73	19	33	125
		Expected Count	70.6	21.3	33.1	125.0
	Superficial	Count	115	16	54	185
		Expected Count	104.5	31.5		185.0
Total		Count	226	68	106	400
		Expected Count	226.0	68.0	106.0	400.0

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	65.813		

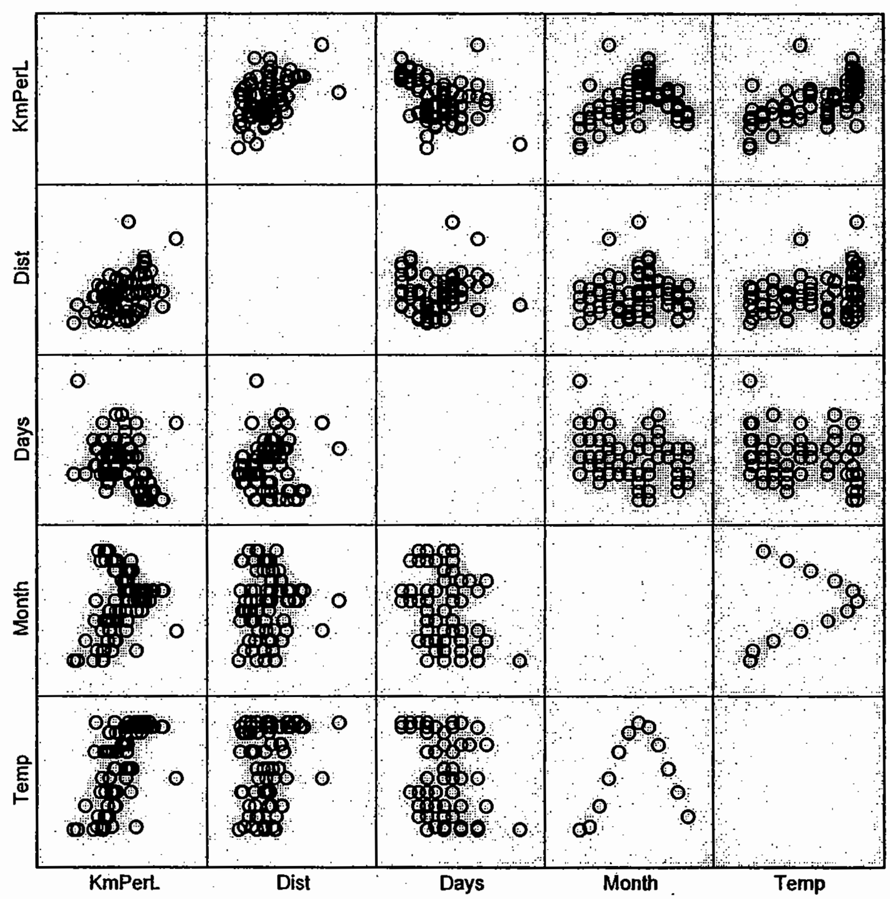
Question 4. Fuel

Case Summaries(a)

	Dist	KmPerL	Days	Month	Temp
1	313.00	6.01	7.00	4.00	5.80
2	374.00	6.55	6.00	4.00	5.80
3	272.00	6.59	5.00	4.00	5.80
4	232.00	7.81	3.00	4.00	5.80
5	268.00	6.57	6.00	5.00	12.50
6	141.00	6.38	3.00	5.00	12.50
7	361.00	7.11	6.00	5.00	12.50
8	141.00	5.44	4.00	5.00	12.50
9	203.00	6.15	4.00	5.00	12.50
10	193.00	7.88	3.00	5.00	12.50
11	214.00	6.79	3.00	5.00	12.50
12	156.00	7.78	5.00	6.00	17.30
13	229.00	6.54	3.00	6.00	17.30
14	187.00	6.58	3.00	6.00	17.30

15	193.00	8.98	3.00	6.00	17.30
16	286.00	8.61	5.00	6.00	17.30
17	167.00	7.05	3.00	6.00	17.30
18	151.00	7.31	5.00	6.00	17.30
19	388.00	8.78	7.00	7.00	19.80
20	195.00	6.96	3.00	7.00	19.80
Total N	20	20	20	20	20

a Limited to first 20 cases.



Error SS = 184.796, df = 85

Dependent Variable: KmPerL

Parameter	B	Std. Error	t	Sig.
Intercept	7.539	.159	47.416	.000

Error SS = 106.080, df = 83

Dependent Variable: KmPerL

Parameter	B	Std. Error	t	Sig.
Intercept	6.409	.415	15.460	.000
Dist	.008	.001	5.883	.000
Days	-.253	.044	-5.757	.000

Error SS = 65.417, df = 72

Dependent Variable: KmPerL

Parameter	B	Std. Error	t	Sig.
Intercept	5.678	.544	10.432	.000
Dist	.004	.001	3.234	.002
Days	-.111	.051	-2.192	.032
[Month=1.00]	-.878	.623	-1.408	.163
[Month=2.00]	.549	.608	.902	.370
[Month=3.00]	.191	.580	.329	.743
[Month=4.00]	1.131	.622	1.820	.073
[Month=5.00]	.423	.559	.756	.452
[Month=6.00]	1.421	.561	2.533	.013
[Month=7.00]	1.696	.529	3.209	.002
[Month=8.00]	2.201	.535	4.118	.000
[Month=9.00]	1.371	.662	2.071	.042
[Month=10.00]	1.370	.581	2.357	.021
[Month=11.00]	.575	.559	1.028	.308
[Month=12.00]	0(a)	.	.	.

a This parameter is set to zero because it is redundant.

Error SS = 79.313, df = 82

Dependent Variable: KmPerL

Parameter	B	Std. Error	t	Sig.
Intercept	5.849	.376	15.556	.000
Dist	.006	.001	4.655	.000
Days	-.135	.044	-3.057	.003
Temp	.069	.013	5.261	.000

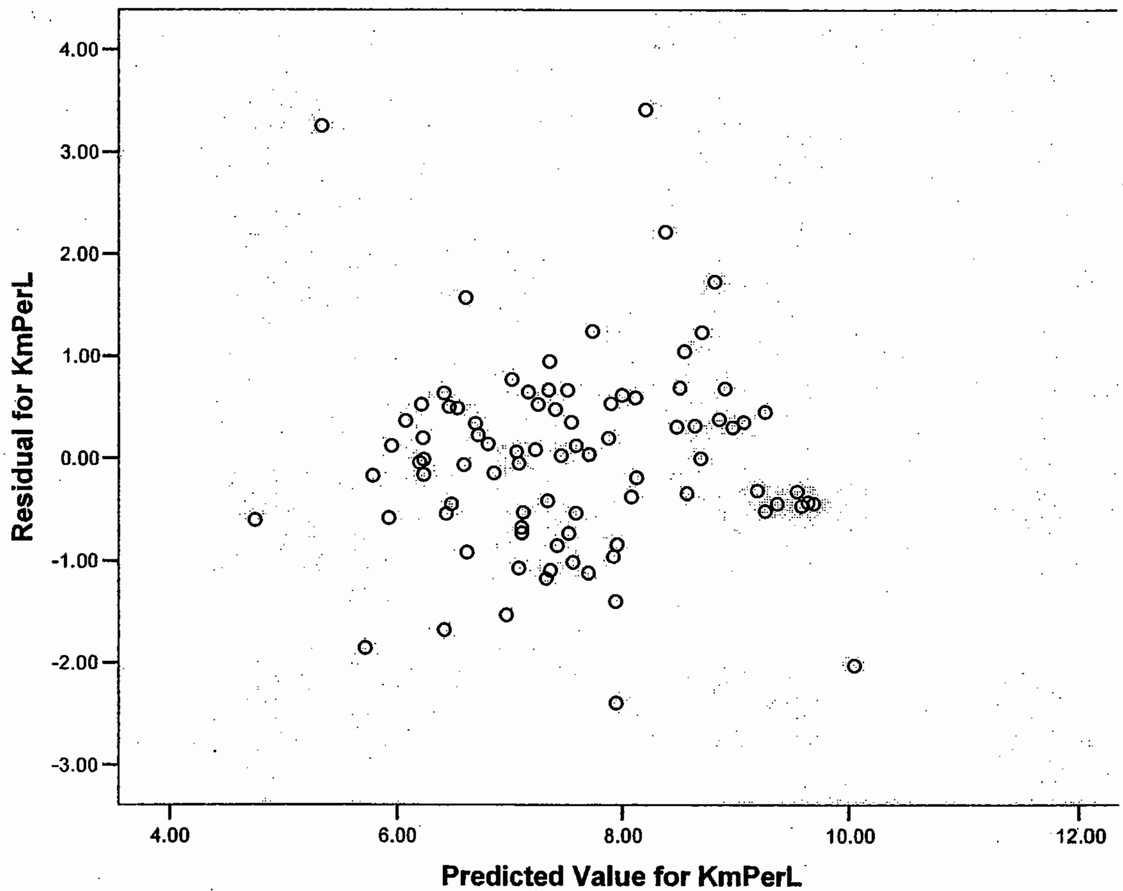
Error SS = 65.417, df = 72

Dependent Variable: KmPerL

Parameter	B	Std. Error	t	Sig.
Intercept	6.036	.427	14.131	.000
Dist	.004	.001	3.234	.002
Days	-.111	.051	-2.192	.032
Temp	.094	.092	1.028	.308
[Month=1.00]	-.567	.819	-.692	.491
[Month=2.00]	.794	.761	1.043	.301
[Month=3.00]	-.054	.489	-.111	.912

[Month=4.00]	.227	.772	.294	.770
[Month=5.00]	-1.113	1.253	-.888	.378
[Month=6.00]	-.566	1.671	-.339	.736
[Month=7.00]	-.527	1.877	-.281	.780
[Month=8.00]	.082	1.781	.046	.963
[Month=9.00]	-.334	1.463	-.228	.820
[Month=10.00]	.240	.916	.262	.794
[Month=11.00]	0(a)	.	.	.
[Month=12.00]	0(a)	.	.	.

a This parameter is set to zero because it is redundant.



Quantiles of the χ^2 distribution

Degrees of freedom	Upper tail probability									
	0.995	0.990	0.975	0.950	0.900	0.100	0.050	0.025	0.010	0.005
1	0.00	0.00	0.00	0.00	0.02	2.71	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21	10.60
3	0.07	0.11	0.22	0.35	0.58	6.25	7.81	9.35	11.34	12.84
4	0.21	0.30	0.48	0.71	1.06	7.78	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	0.68	0.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.95
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
35	17.19	18.51	20.57	22.47	24.80	46.06	49.80	53.20	57.34	60.27
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
45	24.31	25.90	28.37	30.61	33.35	57.51	61.66	65.41	69.96	73.17
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
55	31.73	33.57	36.40	38.96	42.06	68.80	73.31	77.38	82.29	85.75
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
80	51.17	53.54	57.15	60.39	64.28	96.58	101.88	106.63	112.33	116.32
100	67.33	70.06	74.22	77.93	82.36	118.50	124.34	129.56	135.81	140.17
120	83.85	86.92	91.57	95.70	100.62	140.23	146.57	152.21	158.95	163.65

Quantiles of the F distribution, $P = 0.05$

Denominator degrees of freedom	Numerator degrees of freedom 1	2	3	4	5	6	7	8	9	10
1	161.4	199.5	215.7	224.6	230.2	233.0	236.8	238.9	240.5	241.9
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
45	4.06	3.20	2.81	2.58	2.42	2.31	2.22	2.15	2.10	2.05
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03
55	4.02	3.16	2.77	2.54	2.38	2.27	2.18	2.11	2.06	2.01
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

Critical values of T_L and T_U for the Wilcoxon rank sum test: Independent samples. Test statistic is the rank sum associated with the smaller sample.

$P = 0.025$, one tailed, $P = 0.05$ two tailed.

n_1	3		4		5		6		7		8		9		10	
n_2	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U
3	5	16	6	18	6	21	7	23	7	26	8	28	8	31	9	33
4	6	18	11	25	12	28	12	32	13	35	14	38	15	41	16	44
5	6	21	12	28	18	37	19	41	20	45	21	49	22	53	24	56
6	7	23	12	32	19	41	26	52	28	56	29	61	31	65	32	70
7	7	26	13	35	20	45	28	56	37	68	39	73	41	78	43	83
8	8	28	14	38	21	49	29	61	39	73	49	87	51	93	54	98
9	8	31	15	41	22	53	31	65	41	78	51	93	63	108	66	114
10	9	33	16	44	24	56	32	70	43	83	54	98	66	114	79	131

$P = 0.05$, one tailed, $P = 0.10$ two tailed.

n_1	3		4		5		6		7		8		9		10	
n_2	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U	T_L	T_U
3	6	15	7	17	7	20	8	22	9	24	9	27	10	29	11	31
4	7	17	12	24	13	27	14	30	15	33	16	36	17	39	18	42
5	7	20	13	27	19	36	20	40	22	43	24	46	25	50	26	54
6	8	22	14	30	20	40	28	50	30	54	32	58	33	63	35	67
7	9	24	15	33	22	43	30	54	39	66	41	71	43	76	46	80
8	9	27	16	36	24	46	32	58	41	71	52	84	54	90	57	95
9	10	29	17	39	25	50	33	63	43	76	54	90	66	105	69	111
10	11	31	18	42	26	54	35	67	46	80	57	95	69	111	83	127

Critical values of T_0 for the Wilcoxon paired difference signed rank test.

	n																
One-tailed	Two-tailed	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$P = 0.05$	$P = 0.10$	1	2	4	6	8	11	14	17	21	26	30	36	41	47	54	60
$P = 0.025$	$P = 0.05$		1	2	4	6	8	11	14	17	21	25	30	35	40	46	52

End of exam