

# MATH 556 - ASSIGNMENT 1

To be handed in not later than 11.59pm, 30th September 2022.  
Please submit your solutions as pdf via myCourses.

1. Consider the pdf,  $f_X(x)$ , for continuous random variable  $X$  that takes the form

$$f_X(x) = \mathbb{1}_{(-1,0)}(x)(c+x) + \mathbb{1}_{[0,1)}(x)(c-x) \quad x \in \mathbb{R}$$

for some constant  $c$ , where  $\mathbb{1}_A(x)$  is the indicator function for set  $A$

$$\mathbb{1}_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A. \end{cases}$$

- (a) Find the value of  $c$ . 2 Marks  
(b) Find the form of cdf,  $F_X(x)$ . 3 Marks  
(c) Find the quantile function,  $Q_X(p)$  for  $0 < p < 1$ . 3 Marks  
(d) Find the expected value of the quantity  $|X|$ ,  $\mathbb{E}_X[|X|]$ , defined by

$$\mathbb{E}_X[|X|] = \int_{-\infty}^{\infty} |x|f_X(x) dx$$

2 Marks

*Hint: first sketch the pdf.*

2. Suppose that  $X = (X_1, X_2)$  where  $X_1$  and  $X_2$  are independent continuous random variables where each has a *Uniform*(0, 1) distribution. Find, by integrating the joint pdf over a suitable set  $B$  in  $\mathbb{R}^2$ , the probability

$$P_X[X_1 > 2X_2].$$

*Hint: first identify the set  $B$ .*

4 Marks

3. One fair red die and one fair blue die are rolled, with the results of the rolls independent events. Let  $X_1$  denote the sum of the scores on the two dice, and let  $X_2$  denote the value equal to the score on the red die minus the score on the blue die.

Are  $X_1$  and  $X_2$  independent random variables? Justify your answer.

3 Marks

4. Suppose that  $X$  is a continuous random variable with distribution specified so that

$$P_X[X > x] = \begin{cases} 1 & x < 0 \\ \exp\{2(1 - e^x)\} & x \geq 0 \end{cases}$$

Find the pdf of random variable  $Y$ , where

$$Y = \exp\{X\}.$$

3 Marks