1. Determine the general solution of the PDE

$$(y^2 - u^2)\frac{\partial u}{\partial x} - xy\frac{\partial u}{\partial y} = xu$$

and then determine that solution passing through the line $x = y = u$. (If possible, find an explicit solution for u).

2. An infinitely long thin wire loses heat through its curved surface to the surroundings such that the temperature in the wire evolves according to the PDE

$$u_t = u_{xx} - u, \qquad -\infty < x < \infty.$$

The initial temperature is given by $u(x,0) = f(x)$.

   (a) Use an appropriate transform and convolution theorem to find $u(x,t)$ for $t > 0$;

   (b) Determine $u(x,t)$ in the case $u(x,0) = \delta(x - x_0)$, where δ is the Dirac delta function.

3. Determine the characteristic directions and the ODEs that apply along the characteristics for the system

$$\begin{aligned} u_x + 2e^x v_x + u_y &= 0 \\ 2e^{-x}u_x + v_x + v_y &= 0. \end{aligned}$$

4. If the temperature of a spherical surface $r = a$ is maintained at $u(a,\phi) = 1 + \cos\phi$, determine the steady-state temperature *outside* the sphere if u tends to zero as $r \to \infty$. When there is axial symmetry, Laplace's equation is:

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{\sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial u}{\partial\phi}\right) = 0.$$

5. If the temperature distribution in an insulated circular plate is independent of θ, it is then described by solutions of

$$k\left(u_{rr} + \frac{1}{r}u_r\right) = u_t, \quad 0 < r < a,\ t > 0,\ BC : u_r(a,t) = 0.$$

Solve the initial-value problem if $u(r,0) = f(r)$.

6. Solve the initial-value problem on the infinite domain $-\infty < x < \infty$ consisting of the PDE

$$3u_{xx} - 4u_{xt} + u_{tt} = 0$$

with initial conditions $u(x,0) = \sin x$ and $u_t(x,0) = 0$. (Hint: d'Alembert would have solved this one with no trouble.)

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-586A

APPLIED PARTIAL DIFFERENTIAL EQUATIONS

Examiner: Professor S.A. Maslowe	Date: Thursday, December 10, 1998
Associate Examiner: Professor J. Turner	Time: 1:00 P.M. - 4:00 P.M.

This exam comprises the cover and 1 page of questions.