McGILL UNIVERSITY
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH & STAT 189-488A

Set Theory
Final exam

Examiner: M. Barr — Time: 2:00 P.M.–5:00 P.M.
Associate Examiner: M. Makkai — Date: 96–12–12

Fill in your name and student number in the space below.

Family Name: ______________________________

Given Name(s): ______________________________

Student Number: ______________________________

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This is an open book exam. You may use your notes and book freely. Answer any five questions. They have equal value.

All work is to be done on this examination form. This exam comprises 7 pages, including this cover page.

Show that every ordinal α can be represented in one and only way as $\alpha = \beta + n$, where β is a limit ordinal and $n \in \mathbf{N}$. (Hint: Use transfinite induction, although there are other ways.) (Consider 0 to be a limit ordinal.)

Let $(P, \leq)$ be a poset. Use Zorn's lemma (or otherwise) to show that there is a linear order $\sqsubseteq \subseteq P \times P$ such that $x \leq y$ implies $x \sqsubseteq y$.

A poset is *complete* if every subset has a least upper bound and a greatest lower bound. Suppose C is a complete poset and that P_0 is a subset of the poset P. Show that every order preserving function $f_0 : P_0 \to C$ can be extended to an order preserving function $f : P \to C$.

Let $P(X)$ be a property of sets. Say that P is of *finite character* if $P(X)$ holds if and only if $P(A)$ holds for every finite subset $A \subseteq X$. Show that for all sets X, there is a maximal subset $Y \subseteq X$ for which $P(Y)$ holds.

Show that if Th is an equational theory, $f : M \to N$ a homomorphism between two models of Th and

$$E = \{\langle m_1, m_2 \rangle \in M \times M \ ; \ f(m_1) = f(m_2)\}$$

then E is a congruence on M and that M/E is isomorphic to the submodel of N which is the image of f.

Write down a positive first order theory whose models are posets and show by example that the conclusion of the preceding question is false for posets.