1. One of the following functions [a) or b)] is the real part of an analytic function $f(z) = f(x+iy)$ such that $f(0) = 0$. Find the imaginary part of that function.

   (a) $x^2 + y^2$ , b) $x^2 - y^2$.

2. Let $F(z) = \dfrac{(z+1)}{(z-1)(z^2-4)}$.

   (a) Expand $F(z)$ in a Laurent series valid for $|z| > 2$.

   (b) Expand $F(z)$ in a Laurent series valid for $1 < |z| < 2$.

   (c) Find $\mathbb{Z}^{-1}[F(z)] = f(nT) =$ the inverse $\mathbb{Z}$-transform of $F(z)$. (State explicitly $f(nT)$ in terms of $n$).

3. (a) Find all the values of $1^\pi$.

   (b) Find all the values of $z$ such that $\cos z = i$.

4. Evaluate $\displaystyle\int_0^\infty \frac{dx}{x^{1/4}(x+2)}$. Explain your work. In particular, begin by stating the definition of this improper integral.

5. Let $f(z) = \dfrac{\operatorname{Log} z}{(z^2-1)^2}$, where $\operatorname{Log} z$ is the principal value of $\log z$.

   (a) Find the residue of $f(z)$ at $z = 1$.

   (b) Compute $\displaystyle\int_C f(z)dz$ if $C$ is the circle $|z-1| = 1/2$.

6. Find the Fourier Transform of $f(t) = \dfrac{1}{t^2 - 2t + 5}$, $-\infty < t < \infty$.

7. Find the Inverse Laplace Transform of

$$F(s) = \frac{e^{-2s}}{s(s-3)^2}$$

   by using an appropriate complex line integral.

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATHEMATICS 189-381A

COMPLEX VARIABLES AND TRANSFORMS

Examiner: Professor I. Klemes | Date: Friday, December 20, 1996
Associate Examiner: Professor D. Sussman | Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

**Calculators Not Permitted**
**Answer all 7 questions**
**Each of the 7 questions is worth 10 marks**

This exam comprises the cover and 1 page of questions.