1. (10%) Formulate and prove the three rotations theorem.

2. (10%) Let T be a reflection in the axis m and S any other isometry of E^2.
   What is STS^{-1}? Justify your answer.

3. (20%) Prove that there exists only two types of discrete groups of symmetries of finite figures, namely C_n and D_n.

4. (10%) Give a definition of the symmetric group S_n.
   Let $\sigma = (1527)(436)$ and $\tau = (165)(27)(34)$ in S_8. Compute the product $\tau\sigma$. Write the following permutation
   $$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & 8 & 5 & 6 & 4 & 11 & 10 & 9 & 3 & 1 & 2 \end{pmatrix}$$
   as a product of disjoint cycles.

5. (10%) Define the sphere S^2 and lines of S^2. Prove that if l is a line of S^2 and P is a point, which is not a pole of l, then there is a unique line m through P perpendicular to l. Find m if $l = \{x | < x, \xi >= 0, \xi = (-1/\sqrt{2}, 1/\sqrt{2}, 0)\}$ and $P = (0, 1, 0)$.

6. (10%) Define the projective plane P^2 and the mapping $T : E^2 \to P^2 - l_\infty$.

7. (10%) Define intersecting, parallel and ultraparallel lines of the hyperbolic plane H^2. How does one find the point of intersection of two intersecting lines? If $\xi = (1, -1, 1)$ and $\eta = (0, -1, 0)$, what can you say about l_ξ and l_η?

8. (10%) Prove that the angle sum for a right triangle in H^2 is less than π.

9. (10%) Give a definition of Minkowski space-time. Give the formulas for Lorentz' transformation. What is the light cone?

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-348A

TOPICS IN GEOMETRY

Examiner: Professor O. Kharlampovich
Associate Examiner: Professor K.P. Russell

Date: Monday, December 21, 1998
Time: 9:00 A.M. - 12:00 Noon.

INSTRUCTIONS

Calculators are not permitted.
Answer all questions.

This exam comprises the cover and 1 page of questions.