1. (10%) Formulate and prove the three rotations theorem.

2. (10%) Let $T$ be a reflection in the axis $m$ and $S$ any other isometry of $E^2$. What is $STS^{-1}$? Justify your answer.

3. (20%) Prove that there exists only two types of discrete groups of symmetries of finite figures, namely $C_n$ and $D_n$.

4. (10%) Give a definition of the symmetric group $S_n$.

   Let $\sigma = (1527)(436)$ and $\tau = (165)(27)(34)$ in $S_8$. Compute the product $\tau \sigma$. Write the following permutation

   \[
   \begin{pmatrix}
   1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
   7 & 8 & 5 & 6 & 4 & 11 & 10 & 9 & 3 & 1 & 2
   \end{pmatrix}
   \]

   as a product of disjoint cycles.

5. (10%) Define the sphere $S^2$ and lines of $S^2$. Prove that if $l$ is a line of $S^2$ and $P$ is a point, which is not a pole of $l$, then there is a unique line $m$ through $P$ perpendicular to $l$. Find $m$ if $l = \{x| < x, \xi >= 0, \xi = (-1/\sqrt{2}, 1/\sqrt{2}, 0) \}$ and $P = (0, 1, 0)$.

6. (10%) Define the projective plane $P^2$ and the mapping $T: E^2 \to P^2 - l_\infty$.

7. (10%) Define intersecting, parallel and ultraparallel lines of the hyperbolic plane $H^2$. How does one find the point of intersection of two intersecting lines? If $\xi = (1, -1, 1)$ and $\eta = (0, -1, 0)$, what can you say about $l_\xi$ and $l_\eta$?

8. (10%) Prove that the angle sum for a right triangle in $H^2$ is less than $\pi$.

9. (10%) Give a definition of Minkowski space-time. Give the formulas for Lorentz’ transformation. What is the light cone?
McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-348A

TOPICS IN GEOMETRY

Examiner: Professor O. Kharlampovich
Associate Examiner: Professor K.P. Russell

Date: Monday, December 21, 1998
Time: 9:00 A.M. - 12:00 Noon.

INSTRUCTIONS

Calculators are not permitted.
Answer all questions.

This exam comprises the cover and 1 page of questions.