McGILL UNIVERSITY
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH & STAT 189-338A
History and Philosophy of Mathematics
Final exam

Examiner: M. Barr
Associate Examiner: J. Lambek

Time: 2:00 P.M.–5:00 P.M.
Date: 96–12–18

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Fill in your name and student number in the space below.

Family Name: _____

Given Name(s): _____

Student Number: _____

Answer five questions. If you do six, indicate which five you want to have marked. The questions have equal weight.

This is a closed book examination. Calculators are not permitted.

All work is to be done on this examination form. This exam comprises 13 pages, including this cover page.

Consider the Fibonacci type sequence u_n given by

$$u_0 = 0, \qquad u_1 = 1, \qquad u_{n+1} = u_n + 6u_{n-1}$$

Find an explicit formula for u_n.

Solve the cubic equation $x^3 + 3x^2 - 3x + 1 = 0$ using Cardano's method.

Suppose x and y are two complex numbers such that $xy = 4$ and $x^3 = 4(1 + i\sqrt{3})$. What are the possible values of $x + y$?

Solve the quartic equation

$$x^4 + x^2 + 4x - 3 = 0$$

using Ferrari's method. (Hint: the associated cubic equation has an integer solution, which you can find by trial and error.)

(a) Show that if F is a field and $\sqrt{c} \notin F$ then for any a, $b \in F$, either $a = b = 0$, or $a + b\sqrt{c}$ has in an inverse in $F[\sqrt{c}]$.

(b) What goes wrong with the argument when c has a square root in F.

Begin with the fact that $17^2 + 1 = 10 \cdot 29$ to find a representation of 29 as a sum of two squares. The point of this question is not to show that $29 = 2^2 + 5^2$, but to show that you have understood the argument that any prime of the form $4k + 1$ is a sum of two squares by carrying it out in a specific case.