1. Let $X_1, X_2, \cdots, X_n$ denote a random sample of size n from a distribution with density function

$$f(x) = \begin{cases} \theta x^{\theta-1} & 0 < x < 1,\ \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Obtain an estimator of θ using the method of moments.

   (b) Obtain the maximum likelihood estimator of θ.

2. Let $X_1, X_2, \cdots, X_n$ denote a random sample from a distribution with density function depending on an unknown parameter θ.

   (a) Write down the generalized likelihood ratio test procedure for testing the null hypothesis $H_0 : \theta = \theta_0$ versus the alternative hypothesis $H_1 : \theta \neq \theta_0$. Identify clearly the rejection region.

   (b) Write down the asymptotic distribution of the generalized likelihood ratio statistic in (a) for large sample size n.

   (c) Now suppose that we have n_X observations on a binomial random variable X with parameter p_X and n_Y observations on a binomial random variable Y with parameter p_Y, Y being independent of X.

      i. Write down the generalized ratio statistic.

      ii. Suppose now that $n_X = 100$ and $n_Y = 200$ and that the observed values of X and Y are respectively $x = 40$ and $y = 60$. What do you conclude about the equality of p_X and p_Y?

3. The midterm and final exam scores of 5 students in a statistics course are tabulated below.

Midterm	67	70	64	74	82
Final	73	83	79	91	94

   We will assume that the errors associated with these data are all independently, identically and normally distributed with mean 0 and unknown variance σ^2.

   (a) Compute least-squares estimates of the intercept a and the slope b of a simple linear regression line $y = a + bx$ to fit these data.

   (b) Plot these data and draw in the line you estimated in (a).

   (c) Find a two-sided 95% confidence interval for b.

   (d) Predict the final exam score of a student who obtained 84% on the midterm.

4. Let μ_1, μ_2, μ_3 be the respective means of three independent normal distributions with a common but unknown variance σ^2. In order to test at the $\alpha = 5\%$ significance level the hypothesis $H_0 : \mu_1 = \mu_2 = \mu_3$ against all possible alternative hypotheses, a random sample of size 4 is taken from each of these distributions and the observed values from the three distributions are, respectively

X_1:	5	9	6	8
X_2:	11	13	10	12
X_3:	10	6	9	9

   (a) Determine whether H_0 should be rejected or not.

   (b) Find a 95% confidence interval for the difference $\mu_1 - \mu_3$.

   (c) If the assumption of normality was not satisfied, how would you answer (a)? (Explain what you would do without doing the calculations.)

5. Let us consider an enumerative experiment consisting of n identical trials, where the outcome of each trial falls into one, and only one, of k classes or cells. Let n_i denote the observed number and E_i the expected number of outcomes in class i, $i = 1, \cdots, k$. Let n denote the total number of outcomes and so $n = E_1 + \cdots + E_k = n_1 + \cdots + n_k$.

   (a) Write down a formula for the chi-squared test statistic χ^2 which may be used to test how close the observed n_i are to the expected E_i.

   (b) Write down a formula for the approximate (null) distribution of the statistic χ^2 in (a).

   (c) Show that when $k = 2$ and p_1 is the probability of outcome i, the chi-squared test statistic χ^2 equals
   $$\frac{(n_1 - np_1)^2}{n - p_1(1 - p_1)}.$$

   (d) Write down the asymptotic distribution of the statistic χ^2 in (c).

6. (a) A sample of 360 people was selected and classified according to income and stature with the following results:

		Income		
		Low	Average	High
Stature	Thin	48	40	12
	Average	55	53	29
	Overweight	57	46	20

Test the hypothesis that income and stature are independent at level $\alpha = .05$.

(b) A survey was conducted to determine student, faculty and administrative staff attitudes on a new university parking policy. The distribution of those favoring or opposing the policy was as shown in the following table:

	Student	Faculty	Administration
Favor	252	107	43
Oppose	139	81	40

Do the data support at the level $\alpha = .05$ the hypothesis that the proportion of students, faculty and administrative staff favoring the policy is the same?

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-324B

STATISTICS

Examiner: Dr. M. Dansereau
Associate Examiner: Professor G. Styan

Date: Wednesday, April 28, 1999
Time: 9:00 A.M. - 12:00 Noon.

INSTRUCTIONS

Small hand-held calculators are permitted .
Computations should be correct to 2 decimal places.
Tables of various distributions are provided on pages 3 and 4.

This exam comprises the cover, 3 pages of questions and 3 pages of tables.