1. A handful of cards contains 4 black cards and 4 red cards. Three cards are drawn at random without replacement. Let X be the number of black cards of the three cards drawn.

   (a) What is the name of the distribution of X? The events A, B, C are defined as follows

      A: more black cards than red cards are drawn

      B: at least one black card is drawn

      C: the cards drawn are of the same colour.

   Redefine A, B, C in terms of X and find

   (b) $\Pr(A)$;

   (c) $\Pr(B)$;

   (d) $\Pr(C)$;

   (e) $\Pr(C|A)$.

2. In a bolt factory, machines 1, 2 and 3 respectively produce 20%, 30% and 50% of the total output. Of their output, 5%, 3% and 2% are defective. A bolt is selected at random.

   (a) What is the probability that it is defective?

   (b) Given that it is defective, what is the probability that it was made by machine 1?

3. Workers employed in a large service industry have an average wage of \$7.00 per hour with a standard deviation of \$.50. The industry has 64 workers of a certain ethnic group. These workers have an average wage of \$6.90 per hour. Is it reasonable to assume that the wage rate of the ethnic group is equivalent to that of a random sample of workers from those employed in the service industry?

4. If a random variable X has a moment generating function given by

$$M_X(t) = (0.4e^t + 0.6)^{10},$$

   find $E(X)$ and $\mathrm{Var}(X)$. Also, write down the pmf or pdf of X.

5. The number, N, using a certain computer terminal at the basement of Burnside Hall during time interval of length t between 8:00 pm and 10:00 pm has a Poisson distribution with mean t. Give an expression for the probability that there are exactly 2 users at the computer terminal in a time interval of length t. Do the same if the time interval is of length $3t$.

6. Let (Y_1, Y_2) denote the coordinates of a point chosen at random inside a unit circle whose centre is at the origin. That is, Y_1 and Y_2 have a joint density function given by

$$f(y_1, y_2) = \begin{cases} \frac{1}{\pi}, & y_1^2 + y_2^2 \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Define $R = \sqrt{Y_1^2 + Y_2^2}$ and $\theta = \arctan(Y_1/Y_2)$.

(a) Find the joint density function of (R, θ).

(b) Are R and θ independent? What theorem can be used to justify your conclusion?

7. Let (X, Y) have joint density function given by

$$f(x, y) = \begin{cases} k(1-y), & 0 < x < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find

(a) k;

(b) marginal distribution of Y;

(c) the conditional distribution of X given $Y = y$ for $0 < y < 1$;

(d) the marginal distribution of X;

(e) $\text{Cov}(X, Y)$.

8. Let X and Y be continuous random variables having a joint density function.

(a) Write down the conditional expectation $\text{E}[Y|X = x]$ in terms of the joint density function.

(b) Suppose that $\phi(x)$ is a univariate function. Show that

$$\text{E}[\phi(X)Y] = \int_{-\infty}^{\infty} \phi(x)\text{E}[Y|X = x] f_X(x) dx,$$

where $f_X(x)$ is the marginal density function of X.

9. Let A, B, C be any three points not on a straight line in a plane. Suppose that P is uniformly distributed inside the triangle formed by A, B, C. Connect AP and extend it to intersect BC at D. Show that D is uniformly distributed on the line segment BC.