MARKS

(17) 1. Solve and interpret physically:

(a) $\nabla^2\psi(x,y) = 0; \quad 0 < x < \pi,\ 0 < y < \pi$

(i) $\psi(x,0) = 0$, (ii) $\psi(0,y) = 0$, (iii) $\psi(\pi,y) = 0$, (iv) $\psi(x,\pi) = 5$.

Leave your answer in simplest form.

(b) $\dfrac{\partial\psi}{\partial t} = \nabla^2\psi;\ 0 < x < \pi,\ 0 < y < \pi,\ t > 0.$

(i) $\psi(x,0,t) = 0$, (ii) $\psi(0,y,t) = 0$, (iii) $\psi(\pi,y,t) = 0$, (iv) $\psi(x,\pi,t) = 5$,
(v) $\psi(x,y,0) = f(x,y)$.

(c) $\nabla^2\psi(x,y) = -F(x,y)$.

(i) $\psi(x,0) = 0$, (ii) $\psi(0,y) = 0$, (iii) $\psi(\pi,y) = 0$, (iv) $\psi(x,\pi) = 5$.

(11) 2. (a) Find the eigenvalues and eigenfunctions of

$$x^2y'' + xy' + 3y = \lambda y; \quad y(1) = 0,\ y(2) = 0.$$

(b) Expand $f(x)$, piecewise smooth, in terms of the eigenfunctions of (a).

Leave your answer in simplest form.

(16) 3. Solve and interpret physically:

(a) $\dfrac{\partial\psi}{\partial t} = \dfrac{\partial^2\psi}{\partial x^2}; \quad 0 < x < \pi,\ t > 0.$

(i) $\psi(0,t) = 0$, (ii) $\psi_x(\pi,t) = \left[\dfrac{\partial\psi}{\partial x}(x,t)\right]_{x=\pi} = 0$, (iii) $\psi(x,0) = f(x)$.

(b) $\dfrac{\partial\psi}{\partial t} = \dfrac{\partial^2\psi}{\partial x^2} + h(x,t); \quad 0 < x < \pi,\ t > 0.$

(i) $\psi(0,t) = 0$, (ii) $\psi_x(\pi,t) = \left[\dfrac{\partial\psi}{\partial x}(x,t)\right]_{x=\pi} = 0$, (iii) $\psi(x,0) = f(x)$.

(c) $\dfrac{\partial\psi}{\partial t} = \dfrac{\partial^2\psi}{\partial x^2} + h(x,t).$

(i) $\psi(0,t) = F(t)$, (ii) $\psi_x(\pi,t) = G(t)$, (iii) $\psi(x,0) = f(x)$.

(15) 4. (a) Obtain the general solution of Laplace's equation in spherical coordinates with no θ dependence, i.e. $\nabla^2\psi(r,\phi) = 0,\ 0 \le \phi \le \pi$, with ψ finite at $\phi = 0$ and $\phi = \pi$.

(b) A sphere of radius "a" centered at the origin is placed in a uniform flow with speed V_0 along the z-axis. Find the velocity potential.
Hint: Solve $\nabla^2\psi(r,\phi) = 0,\ r > a, \quad 0 \le \phi \le \pi$.

$$\text{(i)} \left[\frac{\partial\psi}{\partial r}\right]_{r=a} = 0, \qquad \text{(ii)} \lim_{r\to\infty}[\psi(r,\phi) - V_0 r\cos\phi] = 0.$$

(13) 5. A sphere of radius b has its surface maintained at a temperature T_0. There is a constant heat generation at the rate Q. The initial temperature is $f(r)$. Find the temperature at any point inside the sphere after time t.

Leave your answer in simplest form.

Hints: (a) $\psi = \psi(r,t)$, (b) $\dfrac{1}{\alpha^2}\dfrac{\partial\psi}{\partial t} - \nabla^2\psi = \dfrac{Q}{K}$.

(13) 6. Solve and interpret physically:

$$\frac{\partial\psi}{\partial t} = \frac{\partial^2\psi}{\partial x^2} + 6x; \quad 0 < x < 1,\ t > 0$$

$$\text{(i) } \psi(0,t) = 3, \qquad \text{(ii) } \left[\frac{\partial\psi}{\partial x}(x,t)\right]_{x=1} = -2[\psi(1,t) - 5], \qquad \text{(iii) } \psi(x,0) = f(x).$$

Leave your answer in simplest form.

(20) 7. Solve and interpret physically:

(a) $\nabla^2\psi(r,z) = 0; \quad 0 < r < b,\ 0 < z < \pi$.

(i) $\psi(r,0) = 0$, (ii) $\psi(r,\pi) = f(r)$, (iii) $\psi(b,z) = g(z)$.

(b) $\nabla^2\psi(r,z) = -F(r,z); \quad 0 < r < b,\ 0 < z < \pi$.

(i) $\psi(r,0) = 0$, (ii) $\psi(r,\pi) = f(r)$, (iii) $\psi(b,z) = g(z)$.

Good Luck!

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-319B

PARTIAL DIFFERENTIAL EQUATIONS

Examiner: Professor C. Roth	Date: Thursday, April 24, 1997
Associate Examiner: Professor D. Sussman	Time: 2:00 P.M. - 5:00 P.M.

This exam comprises the cover, 2 pages of questions and 1 page of useful information.