McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATH 315

ORDINARY DIFFERENTIAL EQUATIONS

Examiner: Professor J. Hurtubise
Associate Examiner: Professor G. Schmidt

Date: Monday December 17, 2007
Time: 2:00 PM TO 5:00 PM

INSTRUCTIONS

Answer questions in the exam booklets provided.
Calculators are not permitted.
This is a closed book exam.
Use of a regular and or translation dictionary is not permitted.

This exam consists of the cover page, one page of questions and 1 page of the table of Elementary Laplace Transforms.

MATH 315, FINAL EXAMINATION

December 17th, 2007

1. Solve the following differential equations, finding the general solution, and the particular solution corresponding to the initial conditions, if given:

a. (10 pts) $y' = 2y^2 + xy^2, y(0) = 1$

b. (10 pts) $(1 + x^2)y' + 4xy = (1 + x^2)^{-2}$

c. (10 pts) $(1 - x)y'' + xy' - y = 2(x - 1)^2 e^{-x}$, given that e^x solves the homogeneous equation.

d. (10 pts) $dx + y^{-1}(x - sin(y))dy = 0$

e. (10 pts) $y''' - y'' - y' + y = 0$

f. (10 pts) $y'' - 4y' + 4y = 2x + 4e^{2x} + sin(2x)$

2. Solve in a series centred at $x = 0$. You must first decide whether to use a regular power series or a Frobenius series. Discuss the radius of convergence of the solutions.

a. (10 pts) $(1 - x^2)y'' - 2xy' + 2y = 0,$

b. (10 pts) $2x^2y'' - xy' + (1 + x)y = 0.$

3. a. (5 pts) Compute the inverse Laplace transform of

$$\frac{(s-2)e^{-s}}{s^2 - 4s + 3}.$$

b. (5 pts) Compute the Laplace transform of the square wave f of period 2.

$$\begin{aligned} f(t) &= 1, \quad 0 \le t < 1 \\ &= 0, \quad 1 \le t < 2 \\ &= f(t-2), \quad 2 \le t. \end{aligned}$$

c. (10 pts) Solve the initial value problem

$$y'' + 2y' + 3y = sin(x) + \delta(x - 3\pi), \quad y(0) = 0,\ y'(0) = 0$$

Solve the same problem, but with initial conditions $y(0) = 1,\ y'(0) = 0$.

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}, \quad s > 0$
2. e^{at}	$\frac{1}{s-a}, \quad s > a$
3. t^n; $n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
4. t^p, $p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5. $\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$
6. $\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$
7. $\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a\rvert$
8. $\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a\rvert$
9. $e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
10. $e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
11. $t^n e^{at}$, $n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$
14. $e^{ct}f(t)$	$F(s-c)$
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
19. $(-t)^n f(t)$	$F^{(n)}(s)$