1. Using Gaussian elimination, bring the matrix $A$ to reduced row echelon form while keeping track of a book-keeping matrix:

$$A = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 \\
3 & 4 & 5 & 6 & 7 \\
4 & 5 & 6 & 7 & 8
\end{pmatrix}$$

(a) Find all the dependence relations on the rows of $A$.
(b) Find a basis for the row space of $A$.
(c) Find a basis for the column space from among the columns of $A$.
(d) Find a basis for the column null space.
(e) Is the vector $(1, -3, 0)^t$ in the column space of $A$? If it is, display it as a linear combination of the basis found in (c). If not, why not?
(f) Is the vector $(1, -1, 2, 3)$ in the row space? If it is, display it as a linear combination of the basis found in (b). If not, why not?

2. Prove the following theorems.

(a) State the axioms for the determinant function and, using them, prove that if two rows are equal, then the value of the determinant is zero.

(b) Give the definition of a Hermitian matrix $A$. Show that the eigenvalues of a Hermitian matrix $A$ are real.

3. Consider the system of linear differential equations $\mathbf{v}' = A\mathbf{v}$ where

$$A = \begin{pmatrix}
3 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 3
\end{pmatrix}$$

Notice that 3 is an eigenvalue of $A$.

(a) Find the eigenvalues and eigenvectors of $A$.
(b) Find a fundamental matrix for the differential equation.
(c) Using this fundamental matrix, calculate the exponential $e^{At}$
(d) Solve the non-homogeneous system $\mathbf{w}' = A\mathbf{w} + e^{-t} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ with the initial condition $\mathbf{w}(0) = \mathbf{0}$. The answer may be left in the form of an integral.

4. Consider the quadratic form

$$2x^2 - 4xy + 5y^2$$

(a) Using a change of basis (display the transition matrix $P$), diagonalise and classify this quadratic form.
(b) Hence or otherwise, evaluate the double integral

$$\int_0^\infty \int_0^\infty e^{-(2x^2-4xy+5y^2)} \, dx \, dy.$$
McGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATHEMATICS 189-270A

APPLIED LINEAR ALGEBRA

Examiner: Professor W. Jonsson
Associate Examiner: Professor S.A. Maslowe

Date: Monday, December 7, 1998
Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

FACULTY STANDARD CALCULATORS PERMITTED

This exam comprises this cover and 1 page of questions.