1. (10 points) Calculate the line integral,

$$\int_C \frac{-(y-1)}{(x-2)^2 + (y-1)^2} \, dx + \frac{x-2}{(x-2)^2 + (y-1)^2} \, dy$$

where

- (a) C is the circle of radius 1 centered at (2, 1), oriented counterclockwise.
- (b) C is the rectangle with vertices at (1,0), (3,0), (3,2) and (1,2) oriented counterclockwise.
- 2. (10 points) Calculate the line integral,

$$\int_{C} 2z \, dx + (2x+z) dy + (3x+2y) dz$$

where C is the curve of intersection of the cylinder $x^2 + z^2 = 1$ with the plane x + 2y + z = 1oriented counterclockwise when viewed from (0, 1, 0).

- 3. (10 points) Use the method of Lagrange multipliers to find the point on the circle $x^2 + (y-1)^2 = 1$ which is closest to (2,0).
- 4. (10 points) Compute the outward flux of the vector field $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ across the surface $S = S_1 \cup S_2$, where S_1 is the portion of the upper sheet of a circular cone of aperture α inscribed into the unit sphere centered at (0,0,0) and S_2 is the corresponding spherical cap, as in the following diagram

5. (10 points) Given the vector field

$$\vec{F} = (x^2 + y^2 + z^2)^{-3/2} \cdot (x, y, z) + \vec{\nabla} \times (\cos x, y^3 \tan xy, z).$$

Compute $\int \int_{S} \vec{F} \cdot d\vec{S}$, where S is the ellipsoid,

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} = 1.$$

6. (10 points) Compute

$$\int \int \int_W z \, dx dy dz,$$

where \boldsymbol{W} is the tetrahedron with vertices:

A(1,6,2), B(0,1,4), C(-1,2,3) and D(0,3,1).

McGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATHEMATICS 189-265A

ADVANCED CALCULUS

Examiner: Professor N. Kamran Associate Examiner: Professor J. Toth Date: Tuesday, December 7, 1999 Time: 9:00 A.M. - 12:00 Noon

INSTRUCTIONS

Calculators are not permitted.

This exam comprises the cover and one page of questions.