NAME (underline family name):

STUDENT NUMBER: **SIGNATURE:**

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATH 263

ORDINARY DIFFERENTIAL EQUATIONS AND LINEAR ALGEBRA

Examiner: G. Schmidt **Date:** Friday, April 22, 2005
Associate Examiner: J. Loveys **Time:** 14:00 PM - 17:00 PM

Instructions

1. Write your name and student number on this examination script.
2. All your answers must be given within this examination booklet. You may use the blank pages for rough work. You can also request extra paper for rough work, not to be handed in.
3. No books, calculators or notes allowed.
4. Answer all questions providing *full justification* for your answers.
5. Your answers may contain expressions that cannot be computed without a calculator.
6. Circle your answers where confusion could arise.
7. This examination booklet consists of this cover and 9 pages of questions.

GOOD LUCK!

Score Table

Problem	Points	Out of
1.		8
2.		8
3.		8
4.		10
5.		12
6.		12
7.		10
8.		15
9.		17
Total:		100

1. (8 marks) Find the solution $y(x)$ of

$$(x^2+1)y'+3xy=6x,\ y(0)=1.$$

2. (8 marks) Find the solution $y(x)$ of

$$xy'=y+xe^{-y/x},\ y(1)=1.$$

3. (8 marks) Solve implicitly

$$xy^3dx+(x^2y^2-1)dy=0,\ y(1)=1.$$

4. (10 marks) Find the general solution $y(x)$ of

$$y'''+4y'=x+\cos 3x.$$

5. (12 marks) Find the solution $y(x)$ of

$$(x-1)y''-xy'+y=2(x-1)^2e^x,\ y(0)=1, y'(0)=1.$$

given that $y_1(x)=x$ and $y_2(x)=e^x$ are solutions of the corresponding homogeneous equations.

6. (12 marks in total) Let $f(t)$ be equal to 2 for $0\le t\le 2$ and equal to t for $t>2$. Use Laplace transforms, and the table which follows, to solve

$$y''+y'=f,\ y(0)=3, y'(0)=4.$$

function $f(t)$	Laplace transform $F(s)$
1	$1/s\ \ (s>0)$
t^n	$n!/s^{n+1}\ \ (s>0)$
e^{at}	$1/(s-a)\ \ (s>a)$
$\sin at$	$a/(s^2+a^2)\ \ (s>0)$
$\cos at$	$s/(s^2+a^2)\ \ (s>0)$
$e^{-at}f(t)$	$F(s+a)$
$H(t-a)$ or $u_a(t)$ $(a\ge 0)$	$e^{-as}/s\ \ (s>0)$
$\delta(t-a)\ \ (a>0)$	e^{-as}
$H(t-a)f(t-a)$ or $u_a(t)f(t-a)$	$e^{-as}F(s)$
$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\ldots-f^{(n-1)}(0)$
$f*g(t)=\int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$

7. (10 marks in total) Let $U=\{(x_1,x_2,x_3)\in \mathbf{R}^3:\ x_1+x_2+x_3=0\}$.
(a) (5 marks) Find the matrix R corresponding to reflection through U.(You may leave your answer as the product of specific matrices and their inverses, without evaluation.)
(b) (3 marks) What are the eigenvalues and eigenvectors of R?
(c) (2 marks) Verify that the matrix R satisfies $R^2=R$.

8. (15 marks in total) Let

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

(a) (4 marks) Given that 1 is an eigenvalue find a basis for the corresponding eigenspace.
(b) (4 marks) Find the other eigenvalue(s) and bases for their eigenspace(s).
(c) (4 marks) Find orthogonal Q and diagonal D such that $A = QDQ^T$.
(d) (3 marks) Find two different matrices X satisfying $X^2 = A$.

9. (17 marks in total) Consider the matrix $A = \begin{pmatrix} -2 & -3 \\ 3 & -2 \end{pmatrix}$ and the corresponding system of differential equations $\mathbf{x}'(t) = A\mathbf{x}(t)$ with $\mathbf{x}(t) = (x_1(t), x_2(t))$.
(a) (4 marks) Find the eigenvalues and corresponding eigenvectors of A.
(b) (4 marks) Find a basis for the space of real solutions of the system of differential equations.
(c)(3 marks) Find the solution of the system of differential equations satisfying the initial conditions $x_1(0) = 1,\ x_2(0) = -1$.
(d) (3 marks) Write down an expression for e^{At} and use this to write down the general solution of the system of differential equations.
(e) (3 marks) Describe the behaviour of the solutions as $t \longrightarrow \infty$.