1. Use the power series for e^x to obtain a series for $\int_0^1 \frac{e^x - 1}{x} dx$.

   Write out the first 4 terms, and explain how you would estimate the error in using only these terms for the whole sum.

2. Find the equation of the tangent plane and parametric equations for the normal line to the surface with equation $x^3y + y^3z + z^3x = 5$ at $(-2, -1, 1)$.

3. Use the value of $f(x, y) = \ln(x + 2y - 2xy)$ at $(1, 1)$ to find approximately the value of $f(0.9, 1.2)$, using the linear approximation.

4. The temperature of a metal plate as a function of position is given by

$$T(x, y) = \frac{xy}{1 + x^2 + y^2} \ .$$

   (a) Find $\frac{dT}{ds}$ in the direction of the vector $\vec{u} = \vec{\imath} - 2\vec{\jmath}$.

   (b) Find a unit vector in the direction in which $\frac{dT}{ds}$ is greatest.

5. Find and classify the critical points of

$$f(x, y) = 2y^2x - yx^2 + 4xy \ .$$

6. Evaluate $\int\int_D \frac{1}{(1 + x + y)^2} \, dA$ if D is the triangular region with vertices at $(0, 0)$, $(1, 1)$ and $(2, 0)$.

7. Evaluate $\int\int\int_V \cos(x + y + z) dV$ where V is the solid bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y + z = \frac{\pi}{2}$.

8. For the parametric curve $x = 2t$, $y = t^2$, $z = \ln t$ find, as functions of t,

   (a) (i) the velocity, acceleration and speed; (ii) the radius of curvature; (iii) the unit tangent and normal vectors $\vec{T}$ and $\vec{N}$.

   (b) Find parametric equations for the tangent line to the curve in part (a) at the point with $t = 1$.

McGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATHEMATICS 189-260B

INTERMEDIATE CALCULUS

Examiner: Professor D. Sussman
Associate Examiner: Professor N.G.F. Sancho

Date: Monday, April 19, 1999
Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

Faculty standard calculators are permitted.

This exam comprises the cover and 1 page of questions.