MARKS

(12) 1. A particle moves on the surface $\frac{x^2}{2}+yz-\frac{z^2}{2}=2$. At the point $(-2,1,2)$ x is changing at the rate of 3m./sec. and z is changing at the rate of -2m./sec. Determine in m./sec. the rate of change (with respect to time) of $\frac{\partial z}{\partial x}$ at $(-2,1,2)$.

(7) 2. If $\int_{a(x)}^{b(y)} F(x,y,z,t)dt=0$ find $\frac{\partial z}{\partial x}$ in terms of $a(x)$, $b(y)$ and F.

(12) 3. Find the surface area cut from the sphere $x^2+y^2+z^2=36$ by the cylinder $x^2+y^2-6y=0$. Do this problem both in cartesian (and then polars), as well as spherical coordinates.

(11) 4. Find the force of gravitational attraction exerted on a unit mass placed at the origin by the SOLID right circular cone $z^2=3(x^2+y^2)$ bounded by the xy plane and the plane $z=14$, if the density is constant.

(11) 5. A particle is attracted toward the origin by a force proportional to the third power of the distance from the origin. How much work is done in moving the particle from the origin to the point $(1,1)$ along the path $y=x^2$ if the coefficient of friction between the particle and the path is $\frac{1}{4}$?

(9) 6. Find the work done by the force

$$\overrightarrow{F}=\left[8y+\sqrt{x^4+5}\right]\hat{\imath}+[11x+e^{y^2}]\hat{\jmath}$$

in going once around the astroid $x^{2/3}+y^{2/3}=a^{2/3}$.
Hint: Parametric equations for the astroid are

$$x=a\cos^3 t,\ y=a\sin^3 t,\quad 0\le t<2\pi.$$

(14) 7. A non-zero scalar field ψ is such that $\|\overrightarrow{\nabla\psi}\|^2=7\psi$ and $\overrightarrow{\nabla}\cdot(\psi\overrightarrow{\nabla\psi})=11\psi$. Evaluate $\iint_S \overrightarrow{\nabla\psi}\cdot\hat{n}dS$, where S is the surface of the region in the first octant bounded by $z=\sqrt{x^2+y^2}$, $z=\sqrt{1-x^2-y^2}$, $y=\frac{x}{\sqrt{3}}$ and $y=\sqrt{3}x$; $\hat{n}$ is the unit outward normal. Do this problem in both cylindrical and spherical coordinates.

(11) 8. (a) Verify Stokes' theorem for

$$\vec{F} = -2y\hat{\imath} + xz\hat{\jmath} - yz^2\hat{k}$$

and S is the surface of the paraboloid

$$2z = x^2 + y^2;\ 0 \le z \le 2.$$

(5) (b) Verify Stokes' theorem for $\vec{F}$ as in (a) and the surface being the disc $x^2 + y^2 \le 4,\ z = 2$.

(5) 9. (a) Deduce the equation of continuity for fluid flow

$$\vec{\nabla} \cdot (\delta \vec{v}) + \frac{\partial \delta}{\partial t} = Q$$

where δ is the density, $\vec{v}$ the velocity and Q the rate at which fluid is generated (in gm/cm^3/sec, say). Explain carefully all your conclusions.

(3) (b) If, further, the fluid is irrotational and incompressible, show that the velocity potential ψ satisfies

$$\nabla^2 \psi = \frac{Q}{\delta}.$$

Good Luck!

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-248A

ADVANCED CALCULUS I

Examiner: Professor C. Roth | Date: Thursday, December 18, 1997
Associate Examiner: Professor D. Sussman | Time: 9:00 A.M. - 12:00 Noon

INSTRUCTIONS

Only non-programmable calculators are permitted.

This exam comprises the cover, 2 pages of questions and 1 page of useful information.