1. Let $V = \mathbb{R}^3$. Which of the following sets are subspaces and which are not subspaces? Justify your answer:
   (i) $W_1 = \{(a, b, 0) : a, b \in \mathbb{R}\}$;
   (ii) $W_2 = \{(a, b, c) : a + b + c = 0\}$;
   (iii) $W_3 = \{(a, b, c) : a \geq 0\}$;
   (iv) $W_4 = \{(a, b, c) : a^2 + b^2 + c^2 \leq 1\}$.

2. Which of the following sets of vectors in $\mathbb{R}^3$ are linearly independent?
   (a) $(1, 2, 3), (4, 5, 6), (7, 8, 9)$;
   (b) $(1, 0, 1), (2, 3, 1), (1, -1, 6), (0, 2, 4)$;
   (c) $(1, 1, 1), (1, -1, 1), (2, 3, 1)$;

3. Let
$$u_1 = (1, 1, -1), \quad u_2 = (2, 3, -1), \quad u_3 = (3, 1, -5)$$
and
$$v_1 = (1, -1, -3), \quad v_2 = (3, -2, -8), \quad v_3 = (2, 1, -3).$$
Show that $\{u_1, u_2, u_3\}$ and $\{v_1, v_2, v_3\}$ generate the same vector space.

4. Let $T : V \to U$ be a linear transformation of vector spaces. Show that the image of T is a subspace of U and the kernel of T is a subspace of V. If the kernel of T is $\{0\}$, deduce that $\dim V \leq \dim U$.

5. Let V be the vector space of polynomials of degree $\leq n$. Determine whether or not each of the following is a basis of V.
(i) $\{1, \quad 1 + t, \quad 1 + t + t^2, \quad ..., \quad 1 + t + t^2 + \cdots + t^{n-1} + t^n\}$
(ii) $\{1 + t, \quad t + t^2, \quad t^2 + t^3, \quad ..., \quad t^{n-1} + t^n\}$.

6. Let $T : \mathbb{C} \to \mathbb{C}$ be the map $z \to \overline{z}$ (where $\overline{z}$ denotes the complex conjugate $a - bi$ of $z = a + bi, a, b \in \mathbb{R}$). Show that T is NOT linear if $\mathbb{C}$ is viewed as a vector space over itself, but T **is** linear if $\mathbb{C}$ is viewed as a vector space over $\mathbb{R}$.

7. Find an orthogonal matrix P such that $P^{-1}AP$ is diagonal where
$$A = \begin{pmatrix} 11 & -8 & 4 \\ -8 & -1 & -2 \\ 4 & -2 & -4 \end{pmatrix}.$$
(Hint: -5 is an eigenvalue of the matrix.)

8. (i) Let V be the vector space of $n \times n$ matrices over $\mathbb{R}$. If $B^{(t)}$ indicates the transpose of matrix B, show that
$$\langle A, B \rangle = \mathrm{trace}(B^{(t)}A)$$
defines an inner product on V. (Recall that the trace of a matrix is the sum of the diagonal entries.)

   (ii) Find an orthonormal basis for V.