1. Explain the method of Riemann sums to evaluate the definite integral $\int_a^b f(x)dx$. Illustrate your answer by evaluating $\int_0^1 (1+x)^2 dx$, using Riemann sums. Show that the answer is consistent with the fundamental theorem of calculus.

2. (a) Evaluate the following integrals:

   (i) $\displaystyle\int \frac{dx}{x^2+6x+10}$ (ii) $\displaystyle\int x^2 e^{3x} dx$.

   (b) Derive the trapezoidal rule formula to estimate $\int_a^b f(x)dx$.

3. (a) Evaluate the following integrals:

   (i) $\displaystyle\int \sin^3 x \cos^5 x \; dx$ (ii) $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-4}}$.

   (b) Find the area in the first quadrant bounded by the curves $xy = 1$ and $2x + 2y = 5$.

4. (a) Evaluate the following definite integrals or show divergence:

   (i) $\displaystyle\int_0^1 x \ln x \; dx$ (ii) $\displaystyle\int_0^\infty x^2 e^{-5x} dx$.

   (b) Consider the region in the first quadrant bounded by the curves $y = x^2$ and $x = y^2$. Determine the volume of the solid region formed by rotating this plane region about the y-axis.

5. (a) Graph the polar curve $r = 1 - \sin\theta$ and determine the area enclosed by this curve.

   (b) Determine whether the following series converge. Name the tests you are using.

   (i) $\displaystyle\sum_{n=0}^\infty \frac{(-1)^n}{\sqrt{n+5}}$; (ii) $\displaystyle\sum_{n=1}^\infty \frac{1}{\left(n+\frac{1}{2}\right)^2}$; (iii) $\displaystyle\sum_{n=1}^\infty \frac{(-3)^n}{n^3+1}$.

6. (a) Find the area of the surface formed by rotating the curve $y = \cos x, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$, about the x-axis.

   (b) Find the radius of convergence and the interval of convergence:

   (i) $\displaystyle\sum_{n=0}^\infty \frac{2^n x^n}{(n+2)^2}$; (ii) $\displaystyle\sum_{n=0}^\infty \frac{3^n(x-4)^n}{n+3}$; (iii) $\displaystyle\sum_{n=0}^\infty \frac{(x-2)^n}{(n+1)(n+2)}$.

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McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-141A

CALCULUS II

Examiner: Professor J. Turner
Associate Examiner: Professor W.G. Brown

Date: Monday, December 7, 1998
Time: 9:00 A.M. - 12:00 Noon

INSTRUCTIONS

Calculators may not be used.
Answer any FIVE questions.

This exam comprises the cover and 1 page of questions.