1. Explain the method of Riemann sums to evaluate the definite integral \( \int_a^b f(x)dx \).

Illustrate your answer by evaluating \( \int_0^1 (1 + x)^2 dx \), using Riemann sums. Show that the answer is consistent with the fundamental theorem of calculus.

2. (a) Evaluate the following integrals:
    
    (i) \( \int \frac{dx}{x^2 + 6x + 10} \)  
    (ii) \( \int x^3 e^{3x} \, dx \).

    (b) Derive the trapezoidal rule formula to estimate \( \int_a^b f(x) \, dx \).

3. (a) Evaluate the following integrals:
    
    (i) \( \int \sin^3 x \cos^5 x \, dx \)  
    (ii) \( \int \frac{dx}{x^2 \sqrt{x^2 - 4}} \).

    (b) Find the area in the first quadrant bounded by the curves \( xy = 1 \) and \( 2x + 2y = 5 \).

4. (a) Evaluate the following definite integrals or show divergence:
    
    (i) \( \int_0^1 x \ln x \, dx \)  
    (ii) \( \int_0^\infty x^2 e^{-5x} \, dx \).

    (b) Consider the region in the first quadrant bounded by the curves \( y = x^2 \) and \( x = y^2 \). Determine the volume of the solid region formed by rotating this plane region about the \( y \)-axis.

5. (a) Graph the polar curve \( r = 1 - \sin \theta \) and determine the area enclosed by this curve.

    (b) Determine whether the following series converge. Name the tests you are using.
    
    (i) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+5}} \);  
    (ii) \( \sum_{n=1}^{\infty} \frac{1}{(n + 1)^2} \);  
    (iii) \( \sum_{n=1}^{\infty} \frac{(-3)^n}{n^3 + 1} \).

6. (a) Find the area of the surface formed by rotating the curve \( y = \cos x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \), about the \( x \)-axis.

    (b) Find the radius of convergence and the interval of convergence:
    
    (i) \( \sum_{n=0}^{\infty} \frac{2^n x^n}{(n + 2)^2} \);  
    (ii) \( \sum_{n=0}^{\infty} \frac{3^n (x - 4)^n}{n + 3} \);  
    (iii) \( \sum_{n=0}^{\infty} \frac{(x - 2)^n}{(n + 1)(n + 2)} \).

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McGILL UNIVERSITY
FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-141A

CALCULUS II

Examiner: Professor J. Turner
Associate Examiner: Professor W.G. Brown

Date: Monday, December 7, 1998
Time: 9:00 A.M. - 12:00 Noon

INSTRUCTIONS

Calculators may not be used.
Answer any FIVE questions.

This exam comprises the cover and 1 page of questions.