1. Find the derivative function of

   (a) $f(x) = \cos(x^3)$ (b) $f(x) = \cos^3 x$ (c) $f(x) = \dfrac{x^2 + 3x + 4}{\sin x}$

2. A function $y = f(x)$ is known to satisfy the equation $3xy^2 - 4x^2y + y^3 = 5$. Find y' in terms of x and y.

3. Find all the critical points of the function $f(x) = \sin x - x\cos x$ in the interval $(-\pi, 3\pi)$ and classify them.

4. Find the first and second derivative and all the critical points of the function

   $$f(x) = e^{-x}(x^3 - x^2 + x + 1).$$

   Classify the critical points and sketch the graph. Include the behaviour at $\pm\infty$.

5. What point on the line $3x - 2y = 4$ is nearest to the point $(1, -1)$. (In minimizing a distance, you can minimize the square of the distance.)

6. An open topped square box with a volume of 4 cubic meters is to be constructed using a minimum of material. What should its dimensions be?

7. Use the differential to find good approximations to

   (a) $\sqrt{101}$ (b) $28^{\frac{1}{3}}$ (c) $\log_{10} 1002$

8. Find the following limits if they exist or explain why they do not. (Do *not* use L'Hospital's rule).

   (a) $\lim_{x\to 2} \dfrac{x-2}{x^2-4}$ (b) $\lim_{x\to 0} \dfrac{\sin x}{|x|}$

   (c) $\lim_{x\to\infty} \dfrac{4x^3 - 3x + 1}{5x^3 - 4x^2 + 7}$

9. Find the following limits if they exist or explain why they do not. (Use L'Hospital's rule if appropriate).

(a) $\lim_{x\to 0} \dfrac{e^x - e^{-x}}{x}$ (b) $\lim_{x\to 0} \dfrac{x - \sin x}{x^2}$

(c) $\lim_{x\to 2} \dfrac{x^2 - 2}{x + 3}$

10. Show that the function defined by $f(x) = \arcsin x + \arccos x$ has 0 derivative. What can you infer from this fact?

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FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-139A

Calculus I

Examiner: Professor M. Barr
Associate Examiner: Professor W. Brown

Date: Friday, 10 December 1999
Time: 2: 00 pm. – 5: 00 pm.

INSTRUCTIONS

Answer all questions. Each of the ten questions is worth 10 marks.
This is a closed book examination.
Calculators are not permitted.

This exam comprises the cover and 2 pages of questions.