1. Find the derivative function of
   
   (a) \( f(x) = \cos(x^3) \)  
   (b) \( f(x) = \cos^3 x \)  
   (c) \( f(x) = \frac{x^2 + 3x + 4}{\sin x} \)

2. A function \( y = f(x) \) is known to satisfy the equation \( 3xy^2 - 4x^2y + y^3 = 5 \). Find \( y' \) in terms of \( x \) and \( y \).

3. Find all the critical points of the function \( f(x) = \sin x - x \cos x \) in the interval \((-\pi, 3\pi)\) and classify them.

4. Find the first and second derivative and all the critical points of the function

\[
f(x) = e^{-x}(x^3 - x^2 + x + 1).
\]

Classify the critical points and sketch the graph. Include the behaviour at \( \pm\infty \).

5. What point on the line \( 3x - 2y = 4 \) is nearest to the point \((1, -1)\). (In minimizing a distance, you can minimize the square of the distance.)

6. An open topped square box with a volume of 4 cubic meters is to be constructed using a minimum of material. What should its dimensions be?

7. Use the differential to find good approximations to

\[
(a) \sqrt{101} \quad \frac{1}{28.3} \quad (c) \log_{10} 1002
\]

8. Find the following limits if they exist or explain why they do not. (Do not use L’Hospital’s rule).

\[
(a) \lim_{x \to 2} \frac{x - 2}{x^2 - 4} \quad (b) \lim_{x \to 0} \frac{\sin x}{|x|} \quad (c) \lim_{x \to \infty} \frac{4x^3 - 3x + 1}{5x^3 - 4x^2 + 7}
\]
9. Find the following limits if they exist or explain why they do not. (Use L’Hospital’s rule if appropriate).

(a) \( \lim_{x \to 0} \frac{e^x - e^{-x}}{x} \)  
(b) \( \lim_{x \to 0} \frac{x - \sin x}{x^2} \)

(c) \( \lim_{x \to 2} \frac{x^2 - 2}{x + 3} \)

10. Show that the function defined by \( f(x) = \arcsin x + \arccos x \) has 0 derivative. What can you infer from this fact?

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FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-139A

Calculus I

Examiner: Professor M. Barr
Associate Examiner: Professor W. Brown

Date: Friday, 10 December 1999
Time: 2:00 pm. – 5:00 pm.

INSTRUCTIONS

Answer all questions. Each of the ten questions is worth 10 marks.
This is a closed book examination.
Calculators are not permitted.

This exam comprises the cover and 2 pages of questions.