

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS MATH139

Calculus

Examiner: Professor S. W. Drury

Date: Friday, 12 December 2008

Associate Examiner: Professor W. Brown

Time: 2: 00 pm. – 5: 00 pm.

INSTRUCTIONS

Answer all questions in the booklets provided.  
You are expected to simplify your answers wherever possible.  
This is a closed book examination.  
Calculators are not permitted.

Questions 1 thru 3 are worth 9 points each, questions 4 thru 9 are worth 10 points each.  
The exam will be marked out of 87 points and then scaled to a percentage.

**This exam has 9 questions and 4 pages**

1. (9 points) Find the following limits. You may use L'Hospital's Rule, when appropriate, if you wish.

$$(a) \lim_{x \rightarrow \frac{\pi}{6}} \tan(3x)(\cos(2x) - \frac{1}{2}) \quad (b) \lim_{x \rightarrow \infty} \frac{\ln(7e^{3x} + 2x^5)}{4x + 3}$$

$$(c) \lim_{x \rightarrow 1} \frac{x^3 - x}{x^2 + x - 2}$$

2. (9 points) Find the derivative of each of the following functions

$$(a) f(x) = 2x^4 - x^{-5} \quad (b) f(x) = \frac{3x + 1}{x^2 + 1} \quad (c) f(x) = \ln(e^x - x + 2)$$

3. (9 points) Find the derivative of each of the following functions

$$(a) f(x) = \arctan\left(x^{\frac{1}{2}}\right) \quad (b) f(x) = x^{-x} \quad (c) f(x) = x \cosh(x)$$

4. (10 points) Find all horizontal and vertical asymptotes of the graph of

$$f(x) = \frac{\sqrt{x^6 + 1}}{x^3 - 5x^2 + 4x}$$

For each asymptote that you have found, justify your answer by writing down a limit which implies the existence of the asymptote.

5. (10 points) Consider the equation  $x^2y^5 - x^5y^2 - x^7y + 1 = 0$ .

(i) Find  $f'(1)$  for the solution  $y = f(x)$  satisfying  $f(1) = 1$ .

(ii) Find  $g'(1)$  for the solution  $y = g(x)$  satisfying  $g(1) = -1$ .

6. (i) (5 points) Find all critical points of the function  $f(x) = \frac{2 + x^2}{7 + 4x}$  in the interval  $-1 \leq x \leq 1$  and determine where on  $-1 \leq x \leq 1$  it is increasing and decreasing.

(ii) (5 points) Find the absolute minimum value of  $f(x)$  on the interval  $-1 \leq x \leq 1$ .

7. (i) (4 points) Find the first derivative and second derivative of the function

$$f(x) = (3x^2 - 8x + 3)e^x.$$

(ii) (3 points) Determine where  $f$  is increasing and decreasing.

(iii) (3 points) Determine where  $f$  is concave up and concave down.

8. (10 points) A car travelling east at a constant speed of 80 km. per hour leaves a crossroads at noon. A second car leaves at 1 pm. travelling north at 90 km. per hour. At what rate is the distance between the two cars increasing at 3 pm?

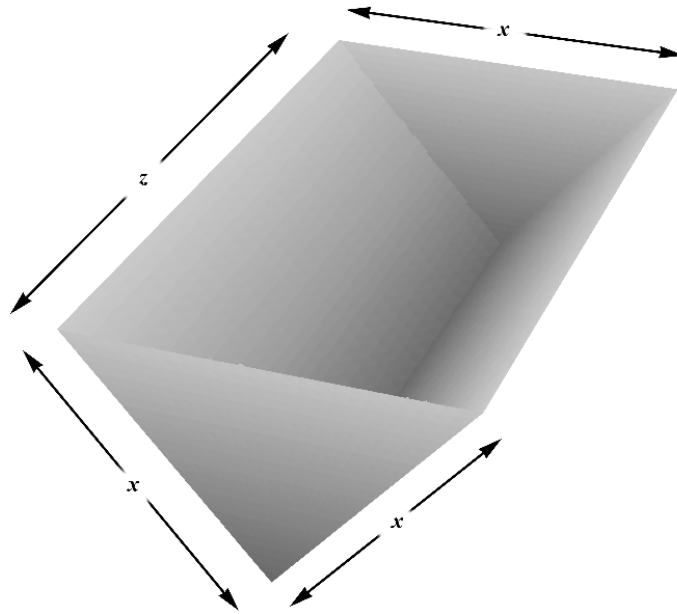


Diagram for Question 9.

9. (10 points) An open trough has an equilateral triangular section, the triangles having side  $x$  units. The length of the trough is  $z$  units. Thus, the surface of the trough consists of two rectangles measuring  $z$  units by  $x$  units and two equilateral triangles with side  $x$  units. Suppose that the volume  $V$  of the trough is fixed and that  $z$  and  $x$  are chosen to minimize the surface area. What is the ratio  $z : x$ ?

\*                    \*                    \*                    \*                    \*