1. (9 marks) Find the following limits. You may use L'Hospital's Rule, when appropriate, if you wish.

$$\text{(a) } \lim_{x\to -3} \frac{x^2+8x+2}{x^2+1} \qquad \text{(b) } \lim_{x\to\infty} \frac{5e^{3x}+9x^9}{8e^{3x}+e^{2x}}$$

$$\text{(c) } \lim_{x\to 0} \frac{\sin(3x)}{\tan(2x)}$$

2. (9 marks) Find the derivative of each of the following functions

(a) $f(x) = 3x^{-2} - 4x^{-5}$ (b) $f(x) = \dfrac{x^5}{x^2+3}$ (c) $f(x) = x^3 \ln(x^2+3x+5)$

3. (9 marks) Find the derivative of each of the following functions

(a) $f(x) = \arctan(x^{-2})$ (b) $f(x) = x^2 e^x$ (c) $f(x) = \sin(\ln(x))$

4. (i) (5 marks) Find all horizontal and vertical asymptotes of the graph of

$$f(x) = \frac{\sqrt{x^2+5}}{x-2}.$$

For each asymptote that you have found, justify your answer by writing down a limit which implies the existence of the asymptote.

(ii) (5 marks) A function g is defined by

$$g(x) = \begin{cases} \sin(x) & \text{if } -\infty < x < 0, \\ ax+b & \text{if } 0 \le x \le \pi/2, \\ \cos(x) & \text{if } \pi/2 < x < \infty. \end{cases}$$

Find the values of a and b that make g continuous on the whole real line.

5. (10 marks) Find the equation of the line tangent to the curve

$$12x + x^2y^3 + 2y^2 = 15$$

at the point $(x, y) = (1, 1)$.

6. (i) (3 marks) Find all the critical points of the function $f(x) = 2\cos(x) + \sin(2x)$ in the interval $0 \leq x \leq \pi$.

   (ii) (4 marks) Classify each such point as a local minimum, a local maximum or some other kind of critical point.

   (iii) (3 marks) Find the absolute maximum value of the function $x \mapsto f(x)$ on the interval $0 \leq x \leq \pi$.

7. (i) (4 marks) Find the first derivative and second derivative of the function

$$f(x) = (x-5)^2 + 8\ln(x)$$

   defined for $x > 0$.

   (ii) (3 marks) Determine where the function is increasing and decreasing.

   (iii) (3 marks) Determine where the function is concave up and concave down.

8. (10 marks) A ladder 13 feet long is standing against a vertical wall. The bottom of the ladder is sliding along the horizontal ground away from the wall at a speed of 1 foot per second. Find the speed of descent of the top of the ladder when the bottom of the ladder is 5 feet from the wall. You should assume that the top of the ladder maintains contact with the wall.

9. (10 marks) A box with a square base and rectangular sides is to be made from material that costs \$5 per square foot for the base, \$4 per square foot for the top and \$3 per square foot for the sides. If the volume of the box is to be 48 cubic feet, what is the minumum cost of material needed for construction.

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FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS MATH139

Calculus I

Examiner: Professor S. W. Drury
Associate Examiner: Professor W. Brown

Date: Friday, 13 December 2002
Time: 2: 00 pm. – 5: 00 pm.

INSTRUCTIONS

Another calculus exam is being written in the same building.
This is the exam for MATH139.
Please make sure that you have the correct exam paper.

Answer all questions.
This is a closed book examination.
Calculators are not permitted.

Questions 1 thru 3 are worth 9 points each, questions 4 thru 9 are worth 10 points each.
The exam will be marked out of 87 points and then scaled to a percentage.

This exam comprises the cover and 2 pages of questions.