1. (9 marks) Find the following limits. You may use L'Hospital's Rule, when appropriate, if you wish.

$$\text{(a) } \lim_{x\to 1} \frac{x^2+10x+1}{x^2+1} \qquad \text{(b) } \lim_{x\to -\infty} \frac{3e^{2x}+4}{2e^{2x}+5}$$

$$\text{(c) } \lim_{x\to 0} \frac{\sin(5x)}{\sin(3x)}$$

2. (9 marks) Find the derivative of each of the following functions

$$\text{(a) } f(x) = 2x^{-5} - 3x^{-7} \qquad \text{(b) } f(x) = \frac{x^3}{x^4+5} \qquad \text{(c) } f(x) = x^4 \ln(x^2+7)$$

3. (9 marks) Find the derivative of each of the following functions

$$\text{(a) } f(x) = \arcsin(x^{-1}) \qquad \text{(b) } f(x) = e^{2x^3} \qquad \text{(c) } f(x) = x\cos(x)$$

4. (10 marks) Find the equation of the line tangent to the curve

$$10x + yx^2 + y^3 + 12 = 0$$

at the point $(x, y) = (-1, -1)$.

5. (i) (3 marks) Find all the critical points of the function $f(x) = 2\sin(x) - \cos(2x)$ in the interval $0 < x < \dfrac{4\pi}{3}$.

(ii) (4 marks) Classify each such point as a local minimum, a local maximum or some other kind of critical point.

(iii) (3 marks) Find the absolute maximum value of the function $x \mapsto f(x)$ on the interval $0 \le x \le \dfrac{4\pi}{3}$.

6. (i) (4 marks) Find the first derivative and second derivative of the function

$$f(x) = (x+1)e^{-x}.$$

(ii) (3 marks) Determine where the function is increasing and decreasing.
(iii) (3 marks) Determine where the function is concave up and concave down.

7. (10 marks) Find the point on the parabola $y = x^2 - 1$ nearest to the point $(2, -\frac{1}{2})$.

Hint : In minimizing a distance, it may be easier to minimize the square of the distance.

8. (10 marks) A box with no top is to be made from a rectangular sheet of cardboard measuring 8 feet by 5 feet by cutting squares of side x feet out of each corner and folding up the sides. What is the largest possible volume of such a box?

9. A function f is defined on the whole real line by

$$f(x) = \begin{cases} x & \text{if } -\infty < x < 0, \\ \pi\cos(x) & \text{if } 0 \le x \le \pi, \\ x - 2\pi & \text{if } \pi < x < \infty. \end{cases}$$

(i) (2 marks) Is f continuous at $x = 0$? If not, what kind of discontinuity does f have at 0?
(ii) (2 marks) Is f continuous at $x = \pi$? If not, what kind of discontinuity does f have at π?
(iii) (2 marks) If it exists, find $\lim\limits_{x\to\pi-} \dfrac{f(x) - f(\pi)}{x - \pi}$, or explain why the limit does not exist.
(iv) (2 marks) If it exists, find $\lim\limits_{x\to\pi+} \dfrac{f(x) - f(\pi)}{x - \pi}$, or explain why the limit does not exist.
(v) (2 marks) Is f differentiable at $x = \pi$?

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FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-139A

Calculus I

Examiner: Professor S. W. Drury
Associate Examiner: Professor W. Brown

Date: Friday, 7 December 2001
Time: 2: 00 pm. – 5: 00 pm.

INSTRUCTIONS

Another calculus exam is being written in the same building.
This is the exam for 189-139A.
Please make sure that you have the correct exam paper.

Answer all questions.
This is a closed book examination.
Calculators are not permitted.

Questions 1 thru 3 are worth 9 points each, questions 4 thru 9 are worth 10 points each.
The exam will be marked out of 87 points and then scaled to a percentage.

This exam comprises the cover and 2 pages of questions.