

189-235A: Basic Algebra I

Assignment 1

Due: Monday, September 23

1. Read Part I of the on-line notes.
2. Do exercise (2) on page 31 of the on-line notes.
3. Do exercise (7) on page 31 of the on-line notes.
4. Do exercise (13) on page 32 of the on-line notes.
5. Use Cardano's formula to solve the following cubic equations. In each case say how many real solutions there are and list all such solutions when there are more than one. (You are advised to use a calculator to check that the expressions you've written down are indeed solutions to the equation at hand.)
 - a. $x^3 + 3x + 1$
 - b. $x^3 - 3x + 1$. (In this case, give a closed form expression for the solution(s) of the equation, in terms of $\cos(2\pi/9)$ and $\sin(2\pi/9)$.)
6. Exercise (11) of page 31 of the on-line notes.
7. Let X be a set, and let $\mathcal{F}(X)$ be the set of all functions from X to itself. This set is equipped with a natural binary operation $(f, g) \mapsto fg$, given by the composition of functions.

- a. Show that $f(gh) = (fg)h$ for all f, g, h in $\mathcal{F}(X)$. (In other words, the operation of composition of functions is always *associative*.)
- b. Show, by providing an example, that fg need not be equal to gf , i.e., that composition of functions *need not be commutative*.
8. Without resorting to a calculator or computer, write the complex number $(1 + i)^{83}$ in the form $a + bi$ where a and b are real numbers.
9. Using the Euclidean algorithm compute the gcd of 123654 and 321456. Show the steps in your calculation.
10. Using induction, show that the addition law in \mathbf{N} is associative directly from the axioms defining addition in \mathbf{N} .
11. Exercise (17), page 32 of the on-line notes.