189-726B: Modular Forms II
Assignment 12

Due: Wednesday, April 16

1. Let E be an elliptic curve over the ring $R = k[\lambda][1/P(\lambda)]$, where $P \in k[\lambda]$ and k is a field. Prove that the Gauss-Manin connection

$$\nabla : H^1_{\mathrm{dR}}(E/R) \longrightarrow H^1_{dR}(E/R) \otimes \Omega^1(R/k)$$

satisfies the Leibniz rule for differentiation:

$$\nabla(f\omega) = \omega \otimes (df) + f\nabla(\omega),$$

for all $f \in R$ and $\omega \in H^1_{\mathrm{dR}}(E/R)$.

2. Let E be the Legendre family

$$y^2 = x(x-1)(x-\lambda)$$

of elliptic curves defined over the ring $R = k[\lambda][\frac{1}{\lambda(\lambda-1)}]$, and let $\omega = \frac{dx}{y}$ be the generator of the module of regular differentials on E. Compute the class

$$\nabla_\lambda \omega = \langle \nabla(\omega), \frac{d}{d\lambda} \rangle,$$

(where the pairing on the right is the natural duality between differential forms and derivations). Express this class as a linear combination, with coefficients in R, of the generators ω and $\eta = x\omega$ for $H^1_{\mathrm{dR}}(E/R)$.