189-726B: Modular Forms II Assignment 8

Due: Wednesday, March 19

1. Let F be an imaginary quadratic field with ring of integers $\mathbf{Z} + \mathbf{Z}\omega$. Complete the proof started in class that

$$\delta_k^r G_k(\omega) = \frac{k(k+1)\cdots(k+r-1)}{(\omega-\bar{\omega})^r} \zeta_F(k+r,-r),$$

where

$$\delta_k^r = \delta_{k+2r-2} \cdots \delta_{k+2} \delta_k$$

is the Shimura-Maass operator,

$$G_k(\tau) = \sum' (m + n\tau)^{-k}$$

is the weight k Eisenstein series, and

$$\zeta_F(m,n) = \sum_{\alpha \in \mathcal{O}_F} \alpha^{-m} \bar{\alpha}^{-n}$$