# 189-726B: Modular Forms II
# Assignment 11

Due: Wednesday, April 9

1. Let $f$ be a $p$-adic modular form of weight $k \in \mathbf{Z}$ with fourier coefficients in $\mathbf{Z}_p$. Show that $f$ can be written (not necessarily uniquely) as

$$f = \sum_{\nu=0}^{\infty} p^{a(\nu)} f_\nu E_{p-1}^{-\nu},$$

where the $f_\nu$ are *classical* modular forms of weight $k + (p-1)\nu$ with fourier coefficients in $\mathbf{Z}_p$, and $a(\nu) \longrightarrow \infty$ as $\nu \longrightarrow \infty$.

More precisely, show that for any ordinary test object $(A, \omega)_{/R}$ defined over a $p$-adic ring $R$ (meaning that the natural map $R \longrightarrow \lim(R/p^n R)$ is an isomorphism), we have

$$f(A, \omega)_{/R} = \sum_{\nu=0}^{\infty} p^{a(\nu)} f_\nu(A, \omega) E_{p-1}(A, \omega)^{-\nu}.$$

2. Let $X$ be a non-singular projective algebraic curve over $\mathbf{C}$ and let $\eta$ be a differential form of the second kind on $X$ over $\mathbf{C}$. Given subsets $U \subset V$ in $X(\mathbf{C})$, with $U$ closed and $V$ open for the usual topology, recall that there exists a smooth function $f_{U,V}$ on $X(\mathbf{C})$ which is identically 1 on $U$ and vanishes outside $V$. Use this fact to give an explicit formula for a smooth differential form $\eta^{an}$ on $X(\mathbf{C})$ representing the same class as $\eta$ in the de Rham cohomology of $X(\mathbf{C})$. Your answer should involve the singularities $P_1, \ldots, P_r$ of $\eta$, mutually disjoint open discs $U_j$ containing $P_j$, and the local primitives $F_j$ of $\eta$ on $U_j$.

3. Let $\omega$ and $\eta$ be differential forms of the first and second kind respectively on $X$ over $\mathbf{C}$. Use the expression for $\eta^{an}$ obtained in part 2 to prove the assertion made in class that:

$$\langle \omega, \eta \rangle = \frac{1}{2\pi i} \int_{X(\mathbf{C})} \omega^{\mathrm{an}} \wedge \eta^{\mathrm{an}},$$

where the pairing on the left is the pairing on algebraic deRham cohomology defined in terms of residues of differential forms.