NOTE TO PRINTER

(These instructions are for the printer. They should not be duplicated.)

THIS EXAMINATION SHOULD BE PRINTED ON $8\frac{1}{2} \times 14$ PAPER, AND STAPLED WITH 3 SIDE STAPLES, SO THAT IT OPENS LIKE A LONG BOOK.

McGILL UNIVERSITY
FACULTY OF SCIENCE
SUPPLEMENTAL/DEFERRED EXAMINATION

MATHEMATICS 189–340B

ABSTRACT ALGEBRA AND COMPUTING

EXAMINER: Professor W. G. Brown — DATE: .....day, August ..th, 2000
ASSOCIATE EXAMINER: Professor W. O. J. Moser — TIME: ..:00 – ..:00 hours

SURNAME: ____________________ SEAT NO.: ___
MR, MISS, MS, MRS, &c.: ____
GIVEN NAMES: ____________________

STUDENT NUMBER: _________ COURSE AND YEAR: _____

Instructions

1. Fill in the above clearly.
2. Do not tear pages from this book; all your writing — even rough work — must be handed in.
3. Calculators are not permitted.
4. This examination booklet consists of this cover, Pages 1 through 8 containing questions; and Pages 9 and 10, which are blank.
5. Show all your work. All solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed!
6. You are advised to spend the first few minutes scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)

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1(a) /5	1(b) /5	2 /10	3(a) /5	3(b) /5	4(a) /5	4(b) /5	5 /10
6(a) /5	6(b) /5	7 /10	8 /10				
				Raw /80	Scaled /100	TERM /30	

1. (a) [5 MARKS] Prove or disprove: *If* $(\mathfrak{B}, *, e)$ *is any non-abelian group, the function* $f : \mathfrak{B} \to \mathfrak{B}$ *defined by* $b \mapsto b^{-1}$ *is not a homomorphism.*

   (b) [5 MARKS] Prove or disprove: *If* $(\mathfrak{A}, *, e)$ *is any group, any homomorphism* $f : (\mathfrak{A}, *, e) \to (\mathfrak{A}, *, e)$ *has the property that the set* $\{a^2 \mid a \in \mathfrak{A}\}$ *is mapped by* f *onto a subgroup of* $(\mathfrak{A}, *, e)$.

2. [10 MARKS] Showing all your work find *all* integer solutions to the system of congruences

$$\begin{aligned} x &\equiv -9 \pmod{3} \\ x &\equiv -8 \pmod{7} \\ x &\equiv -7 \pmod{10} \end{aligned}$$

3. (a) [5 MARKS] Prove or disprove: *If* $(\mathfrak{A}, *, e)$ *is a monoid, there exists a group* $(\mathfrak{B}, \star, e)$ *such that* $a_1 * a_2 = a_1 \star a_2$ *for all* $a_1 \in \mathfrak{A}$ *and* $a_2 \in \mathfrak{A}$, *where* $\mathfrak{A} \subset \mathfrak{B}$ *and* $\mathfrak{B} - \mathfrak{A}$ *consists of exactly one element* b.

(b) [5 MARKS] Prove or disprove: *Let* n *be an even integer,* $n > 4$. *Then there exists a ring containing exactly* n *elements.*

4. (a) [5 MARKS] Prove that the polynomial $x^4 + x^3 + x^2 + x^1 + x^0$ is irreducible over $\mathbb{Z}_2$.

(b) [5 MARKS] Let u be any element — different from both 0 and 1 — of a 16-element field $\mathbb{F}$ over which the polynomial $x^4 + x^3 + x^2 + x^1 + x^0$ has a root. For each of the elements $a \in \mathbb{F}$ determine the product ua, showing all your work.

5. [10 MARKS] Describe 4 groups of order 18, no two of which are isomorphic. You are expected to prove that no two of your groups are isomorphic — by indicating, for every pair of your groups, some property that would be the same for two isomorphic groups, but is not the same for that pair.

6. Let $\mathcal{S}_4$ denote the symmetric group of permutations of the symbols 1, 2, 3, 4, and let $\mathcal{S}_3$ denote the subgroup consisting of all permutations of the symbols 1, 3, 4.

   (a) [5 MARKS] For the element $a = (1234)$ of $\mathcal{S}_4$, determine the right translation $R_a : \mathcal{S}_4 \to \mathcal{S}_4$, expressing it as a permutation of the group elements, in disjoint cycle notation.

   (b) [5 MARKS] Give an example of a right coset of $\mathcal{S}_3$ in $\mathcal{S}_4$ which is not a left coset of $\mathcal{S}_3$ in S_4; or prove that no such example exists.

7. [10 MARKS] Showing all your work, determine *all* integers a and b such that the sum $187\,a + 289\,b$ is equal to the greatest common divisor $(187, 289)$.

8. [10 MARKS] Showing all your work, *carefully* determine all integers n such that $\phi(n) = 10$.

CONTINUATION PAGE FOR PROBLEM NUMBER

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