NOTE TO PRINTER

(These instructions are for the printer. They should not be duplicated.)

THIS EXAMINATION SHOULD BE PRINTED ON $8\frac{1}{2} \times 14$ PAPER, AND STAPLED WITH 3 SIDE STAPLES, SO THAT IT OPENS LIKE A LONG BOOK.

McGILL UNIVERSITY
FACULTY OF SCIENCE
FINAL EXAMINATION

MATHEMATICS 189–340B

ABSTRACT ALGEBRA AND COMPUTING

EXAMINER: Professor W. G. Brown
ASSOCIATE EXAMINER: Professor J. Loveys

DATE: Thursday, April 13th, 2000
TIME: 14:00 – 17:00 hours

SURNAME: SEAT NO.:
MR, MISS, MS, MRS, &c.:
GIVEN NAMES:

STUDENT NUMBER: COURSE AND YEAR:

Instructions

1. Fill in the above clearly.
2. Do not tear pages from this book; all your writing — even rough work — must be handed in.
3. Calculators are not permitted.
4. This examination booklet consists of this cover, Pages 1 through 8 containing questions; and Pages 9 and 10, which are blank.
5. Show all your work. All solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed!
6. You are advised to spend the first few minutes scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)

PLEASE DO NOT WRITE INSIDE THIS BOX

1(a) /5	1(b) /5	2 /10	3(a) /5	3(b) /5	4(a) /4	4(b) /6	5 /10
6(a) /5	6(b) /5	7 /10	8 /5				
				Raw /75	Scaled /100	TERM /30	

1. (a) [5 MARKS] Prove or disprove: If $(\mathfrak{A}, *, e)$ is any non-abelian group, the function $f : \mathfrak{A} \to \mathfrak{A}$ defined by $a \mapsto a^2$ is not a homomorphism.

   (b) [5 MARKS] Prove or disprove: If $(\mathfrak{A}, *, e)$ is any group, any homomorphism $f : (\mathfrak{A}, *, e) \to (\mathfrak{A}, *, e)$ has the property that, for any positive integer k, elements of period (order) k are always mapped onto elements of period (order) k.

2. [10 MARKS] Showing all your work find *all* integer solutions to the system of congruences

$$\begin{aligned} x &\equiv -7 \pmod{3} \\ x &\equiv -8 \pmod{5} \\ x &\equiv -9 \pmod{11} \end{aligned}$$

3. (a) [5 MARKS] Prove or disprove: Any semigroup which is not a monoid is non-commutative.

(b) [5 MARKS] Prove or disprove: If elements x and y of a monoid $(\mathfrak{A}, *, e)$ are both invertible, then $x * y$ is invertible, and

$$(x * y)^{-1} = y^{-1} * x^{-1} .$$

4. (a) [4 MARKS] Prove that the polynomial $x^2 + 2x - 1$ is irreducible over $\mathbb{Z}_3$.

   (b) [6 MARKS] Determine 3 rows of the multiplication table of a field $\mathbb{F}$ of order 9 over which the polynomial $x^2 + 2x - 1$ has a root. (You may choose any 3 rows, but all entries in the rows you select must be shown.)

5. [10 MARKS] Describe 4 groups of order 12, no two of which are isomorphic. You are expected to prove that no two of your groups are isomorphic — by indicating, for every pair of your groups, some property that would be the same for two isomorphic groups, but is not the same.

6. Let $\mathcal{A}_4$ denote the alternating group of even permutations of the symbols 1, 2, 3, 4.

   (a) [5 MARKS] For the element $a = (123)$ of $\mathcal{A}_4$, determine the right translation R_a and the left translation L_a, and express them both as permutations of the group elements, in disjoint cycle notation.

   (b) [5 MARKS] Give an example of a right coset of $\langle(123)\rangle$ in A_4 which is not a left coset of $\langle(123)\rangle$ in A_4; or prove that no such example exists.

7. [10 MARKS] Showing all your work, use the Euclidean algorithm to determine the inverse of $[74]_{53}$ in the group $\mathbb{Z}_{53}^{\times}$.

8. [5 MARKS] Prove or disprove: the ring $\mathbb{Z}_6\{x\}$ of formal power series in an indeterminate x is field.

CONTINUATION PAGE FOR PROBLEM NUMBER

You *must* refer to this continuation page on the page where the problem is printed!

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