

NOTE TO PRINTER

(These instructions are for the printer. They should not be duplicated.)

**THIS EXAMINATION SHOULD BE PRINTED ON $8\frac{1}{2} \times 14$
PAPER, AND STAPLED WITH 3 SIDE STAPLES, SO THAT IT
OPENS LIKE A LONG BOOK.**

McGILL UNIVERSITY — FACULTY OF SCIENCE
SUPPLEMENTAL/DEFERRED EXAMINATION

MATH 329 2004 01 — THEORY OF INTEREST

EXAMINER: Professor W. G. Brown

DATE: XXXXXday, August XX, 2004

ASSOCIATE EXAMINER: Prof. N. Sancho

TIME: XX:00 – XX:00 hours

SURNAME:

MR, MISS, MS, MRS, &c.:

GIVEN NAMES:

STUDENT NUMBER:

INSTRUCTIONS

1. Fill in the above clearly.
2. Do not tear pages from this book; all your writing — even rough work — must be handed in.
3. Calculators. While you are permitted to use a calculator to perform arithmetic and/or exponential calculations, you must not use the calculator to calculate such actuarial functions as $a_{\overline{n}|i}$, $s_{\overline{n}|i}$, $(Ia)_{\overline{n}|i}$, $(Is)_{\overline{n}|i}$, $(Da)_{\overline{n}|i}$, $(Ds)_{\overline{n}|i}$, etc. without first stating a formula for the value of the function in terms of exponentials and/or polynomials involving n and the interest rate. You must not use your calculator in any programmed calculations. If your calculator has memories, you are expected to have cleared them before the examination.
4. This examination booklet consists of this cover, Pages 1 through 8 containing questions; and Pages 9 and 10, which are blank. For all problems you are expected to show all your work, and to simplify algebraic and numerical answers as much as you can. All solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly at the bottom of the page where the question is printed! You may do rough work anywhere in the booklet.
5. You are advised to spend the first few minutes scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)
6. Several useful formulas are printed on page 3. *You should not assume that any of these formulas is/are required in the solution of any of the problems on this examination.*

PLEASE DO NOT WRITE INSIDE THIS BOX

1(a)	1(b)	1(c)	1(d)	2(a)	2(b)	3(a)	3(b)
/3	/3	/3	/3	/5	/5	/7	/8
4(a)	4(b)	4(c)	4(d)	4(e)	5(a)(i)	5(a)(ii)	5(b)
/3	/2	/3	/4	/3	/4	/4	/4
6(a)	6(b)	7	8(a)	8(b)			TOTAL
/8	/7	/8	/5	/8			/100

1. In each of the following problems you are expected to show all your work.
 - (a) [3 MARKS] If $v = 0.95$, determine the value of $d^{(3)}$.
 - (b) [3 MARKS] Showing all your work, determine the nominal interest rate, compounded quarterly, under which a sum of money will double in 10 years.
 - (c) [3 MARKS] Showing all your work, determine the rate of discount, convertible continuously, that is equivalent to a nominal discount rate of 8% per annum, convertible semi-annually.
 - (d) [3 MARKS] If $i^{(\frac{1}{2})} = \frac{1}{30}$, determine the value of $i^{(12)}$.

2. You must show all your work in solving the following problems:
- (a) [5 MARKS] Determine the present value of a perpetuity-immediate of 1000 payable every three months, at an effective annual interest rate of 8%.
 - (b) [5 MARKS] The present value of a perpetuity-immediate paying 100 at the end of every 4 years is $\frac{129,600}{1,105}$. Determine the effective annual interest rate.

Table 1: Several Useful Formulas that you were not expected to memorize

$(Ia)_{\overline{n} i} = \frac{\ddot{a}_{\overline{n} i} - nv^n}{i}$	$(Da)_{\overline{n} i} = \frac{n - a_{\overline{n} i}}{i}$
$(Is)_{\overline{n} i} = \frac{\ddot{s}_{\overline{n} i} - n}{i}$	$(Ds)_{\overline{n} i} = \frac{n(1+i)^n - s_{\overline{n} i}}{i}$
$(Is)_{\overline{n} i} = \frac{s_{\overline{n+1} i} - (n+1)}{i}$	

3. Show detailed work in your solutions to each of these problems.

- (a) [7 MARKS] At a nominal annual interest rate of 6% compounded quarterly, determine the value — 5 years after the last payment — of a decreasing annuity paying 6,000 at the end of the first half-year, 5,500 at the end of the 2nd half-year, and continuing to decrease at 500 per half-year until the final payment of 500.
- (b) [8 MARKS] Three years before the first payment, determine the present value of an annuity that pays 6,000 the first year, 5,900 the second year, with payments continuing to decrease by 100 until it pays 4,000 per year, after which it pays 4,000 forever. The interest rate is 8% effective per year until the first payment of 4,000, after which the interest rate becomes 5% effective forever.

4. The purchase of a new condominium is partially financed by a mortgage of 120,000 payable to the vendor; the mortgage is amortized over 35 years, with a level payment at the end of each half-month, at a nominal annual rate of 9.6% compounded every half-month.
- (a) [3 MARKS] Determine the half-monthly payments under this mortgage.
 - (b) [2 MARKS] Divide the 1st payment into principal and interest.
 - (c) [3 MARKS] Determine the outstanding principal immediately after the 60th payment.
 - (d) [4 MARKS] Divide the 62nd payment into principal and interest.
 - (e) [3 MARKS] The amortization by half-monthly payments was designed to accommodate the purchaser, whose salary was being deposited automatically to his bank account every half-month. The purchaser changes his profession 4 years after the mortgage is executed, and now wishes to make a single payment once every half-year. Determine the amount of that payment if the interest rates are unchanged, but if the mortgage is now amortized to be paid off after 25 years.

5. One of the following equations is always true, and one is true only when $i = 0$.

I. $\frac{1}{s_{\overline{n}|i}} = \frac{1}{a_{\overline{n}|i}} + i$

II. $\frac{1}{s_{\overline{n}|i}} = \frac{1}{a_{\overline{n}|i}} + \frac{1}{i}$

III. $\frac{1}{a_{\overline{n}|i}} = \frac{1}{s_{\overline{n}|i}} + i$

IV. $\frac{1}{a_{\overline{n}|i}} = \frac{1}{s_{\overline{n}|i}} + \frac{1}{i}$

- (a) Explain which is always true, and prove it
- [4 MARKS] algebraically; and
 - [4 MARKS] by a verbal argument, referring to a sinking fund.
(A detailed explanation is expected.)
- (b) [4 MARKS] Prove algebraically that one of the other equations is true for $i = 0$.

6. (a) [8 MARKS] Find the price of the following bond, which is purchased at a premium to yield 5% convertible semi-annually: the bond has face value of 10,000, matures in 12 years at a maturity value of 11,500, and has a nominal coupon rate of 8% per annum, compounded semi-annually; the investor is replacing the premium by means of a sinking fund earning 4% convertible semi-annually.
- (b) [7 MARKS] Suppose that the bond is callable at the end of 9, 10, or 11 years at a premium of 1,000 above the maturity value. Explain what price the investor should pay if she is no longer plans to deposit any of the interest in a sinking fund.

7. [8 MARKS] It was n years ago when James deposited 10,000 in a bank paying 1.8% interest compounded monthly. If he had, instead, placed his deposit in a syndicate paying interest by cheque annually at the rate of 6% per annum, and he had invested only this interest with the bank, how much more interest would he have earned altogether? Show all your reasoning, and express your answer in terms of n .

8. A loan is being repaid with 16 annual payments of 1,000 each. At the time of the 4th payment the borrower requests permission, and is permitted to pay an extra 3000, and then to repay the balance over 8 years with a revised annual payment.

(a) [5 MARKS] If the effective annual rate of interest is 6%, find the amount of the revised annual payment.

(b) [8 MARKS] Complete an amortization table for the last 8 payments, with the following columns:

Payment number	Payment amount	Interest paid	Principal repaid	Outstanding loan balance
5				
...
12				

CONTINUATION PAGE FOR PROBLEM NUMBER

You *must* refer to this continuation page on the page where the problem is printed!

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