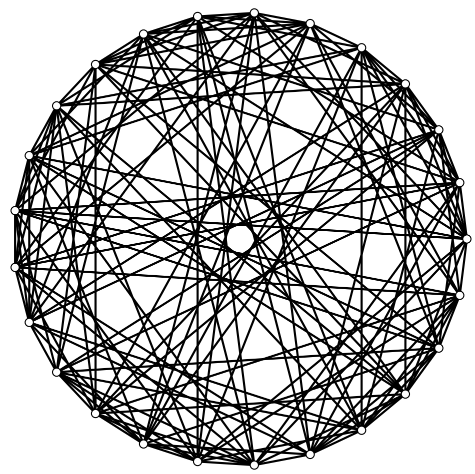
Spectral Graph Theory

WiSe 2015



Module: M.Mat. 4723 – Special Course on Algebraic Structures
Time: Wednesdays 8:30–10
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Trailer: Graph Theory meets Linear Algebra on the street. Graph Theory says: Let X be a graph. Linear Algebra replies: Great. I'll associate some matrices to it, and I'll tell you something about their eigenvalues, maybe I'll even be able to compute them in lots of cases. Graph Theory squints, and asks: OK, but can you also tell me something about X? Can you help me in difficult problems such as colouring or measuring X? Linear Algebra smiles: I can do that, too. Finite Field Theory and Group Theory, overhearing the exchange, join in: You know what? We've got some *really* interesting X's lying around, you should take a look.

Description: The aim of this course is to take this silly dialogue seriously. *Spectral graph theory* is concerned with eigenvalues of matrices associated to graphs, more specifically, with the interplay between spectral properties and graph-theoretic properties. It often feeds on graphs built from groups or finite fields, and this is the direction we will emphasize. In a somewhat larger sense, this course aims to be a sexy introduction to algebraic graph theory. Here, sexy refers to the explicit desire of spicing up the content with lovely applications. For example, we will see several instances where purely combinatorial facts are established via spectral graph theory.

Plan: Graphs: basic notions and examples; invariants (chromatic number, independence number, diameter, girth, isoperimetric constant). Finite fields: basics (extensions, trace and norm); squares (with quadratic reciprocity as a bonus); character sums (from Gauss to Weil). Graphs from finite fields: incidence graphs and Paley graphs. Eigenvalues of graphs: adjacency and laplacian eigenvalues (location, variational formulas); numerous examples (strongly regular graphs, Cayley graphs of abelian groups, spectral computations involving character sums). Bounds: the Alon - Boppana asymptotic threshhold; spectral bounds (for isoperimetric constant, chromatic number, independence number); edge-counting and applications.

Important note: Formally, this course is registered at the master's level. It is, however, a fairly self-contained course that is very much accessible to 3rd year bachelor students.