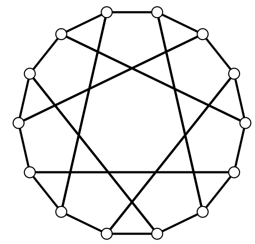
Math 596 - Fall 2017
Spectral graph theory

Instructor: Bogdan Nica



Trailer: Graph Theory meets Linear Algebra on the street. Graph Theory says: Let X be a graph. Linear Algebra replies: Great. I'll associate some matrices to it, and I'll tell you something about their eigenvalues, maybe I'll even be able to compute them in lots of cases. Graph Theory squints, and asks: OK, but can you also tell me something about X? Can you help me in difficult problems such as colouring or measuring X? Linear Algebra smiles: I can do that, too. Finite Field Theory and Group Theory, overhearing the exchange, join in: You know what? We've got some *really* interesting X's lying around, you should take a look.

Description: The aim of this course is to take this silly dialogue seriously. *Spectral graph theory* is concerned with eigenvalues of matrices associated to graphs, more specifically, with the interplay between spectral properties and graph-theoretic properties. It often feeds on graphs built from groups or finite fields, and this is the direction we will emphasize. In a somewhat larger sense, this course aims to be an introduction to *algebraic graph theory*. In an even larger sense, this course aims to braid together several strands of interesting mathematics.

The plan is to cover the following topics:

• GRAPHS: notions and invariants (chromatic and independence numbers, diameter and girth, isoperimetric constant); regular graphs (Cayley and bi-Cayley graphs, strongly regular graphs and design graphs).

• FINITE FIELDS: basics (extensions, trace and norm); squares (with quadratic reciprocity as a bonus); character sums (from Gauss to Weil); graphs from finite fields (incidence graphs and Paley graphs).

• EIGENVALUES OF GRAPHS: adjacency and laplacian eigenvalues (basic properties and examples); computations (strongly regular graphs and combinatorial applications; Cayley graphs of abelian groups and character sums); eigenvalues of symmetric matrices (Courant - Fischer variational formulas, Cauchy interlacing, Weyl's inequality); subgraphs; largest eigenvalues.

• BOUNDS: largest eigenvalues (trees, a spectral Turán theorem); growth of laplacian eigenvalues (and the Alon - Boppana asymptotic threshhold); spectral bounds (for isoperimetric constant, chromatic number, independence number); edge-counting and combinatorial applications.

Prerequisites: A basic familiarity with linear algebra, finite fields, and groups, but not necessarily with graph theory. The course is, however, fairly self-contained and very much accessible to senior undergraduate students.

Grading scheme: 15% attendance and participation + 35% homework + 50 % final exam or written project