Multiple Choice answers

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 1   2   3   4   5   6   7   8   9   10  11  12  13  14  15  16  17  18  19  20

[b,  d,  d,  a,  b,  c,  e,  c,  d,  c,  e,  a,  b,  d,  a,  b,  c,  b,  a,  b]  ver 1

[d,  a,  c,  a,  b,  c,  e,  e,  d,  d,  e,  e,  b,  c,  c,  b,  a,  e,  d,  a]  ver 2

[b,  b,  b,  a,  a,  e,  a,  a,  b,  e,  b,  c,  c,  b,  e,  b,  a,  b,  d,  a]  ver 3

[c,  c,  e,  a,  b,  e,  a,  a,  a,  e,  d,  b,  a,  b,  e,  a,  d,  b,  b,  c]  ver 4
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21. Since $x^3 - 2x - 4$ vanishes when $x = 2$, $x - 2$ divides $x^3 - 2x - 4$. Long division of polynomials gives $x^3 - 2x - 4 = (x - 2)(x^2 + 2x + 2)$. The correct form for partial fractions is then

$$\frac{5x}{x^3 - 2x - 4} = \frac{A}{x-2} + \frac{Bx + C}{x^2 + 2x + 2}$$

and we get

$$5x = A(x^2 + 2x + 2) + (Bx + C)(x - 2)$$

Equating coefficients gives:

$$0 = A + B \qquad 5 = 2A - 2B + C \qquad 0 = 2A - 2C$$

which solves easily to $A = 1$, $B = -1$, $C = 1$. The correct expression in partial fractions is

$$\frac{5x}{x^3 - 2x - 4} = \frac{1}{x-2} - \frac{x-1}{x^2 + 2x + 2}$$

Therefore

$$\int_3^{\infty} \frac{5x}{x^3 - 2x - 4} dx = \int_3^{\infty} \frac{1}{x-2} dx - \int_3^{\infty} \frac{x+1}{x^2 + 2x + 2} dx + \int_3^{\infty} \frac{2}{x^2 + 2x + 2} dx$$

$$= \left[\ln(x-2) - \frac{1}{2} \ln(x^2 + 2x + 1) \right]_3^{\infty} + 2 \int_{\arctan 4}^{\frac{\pi}{2}} d\theta$$

after the substitution $x + 1 = \tan(\theta)$

$$= \lim_{x \to \infty} \ln \left(\frac{x - 2}{\sqrt{x^2 + 2x + 1}} \right) + \frac{1}{2} \ln(17) + \pi - 2\arctan(4)$$

$$= \frac{1}{2} \ln(17) + \pi - 2\arctan(4)$$

Alternative correct answers:

$$\frac{1}{2} \ln(17) + 2\arctan\left(\frac{1}{4}\right), \qquad \frac{1}{2} \ln(17) + \arctan\left(\frac{8}{15}\right), \qquad \frac{1}{2} \ln(17) + \frac{\pi}{2} - \arctan\left(\frac{15}{8}\right)$$

22. The two circles meet when $\cos(\theta) = \sqrt{3}\sin(\theta)$, i.e. when $\tan(\theta) = \frac{1}{\sqrt{3}}$, i.e. when $\theta = \frac{\pi}{6}$

It follows that the area is

$$\frac{1}{2} \int_0^{\frac{\pi}{6}} 3\sin(\theta)^2 d\theta + \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos(\theta)^2 d\theta$$

$$\frac{1}{2} \int_0^{\frac{\pi}{6}} 3\sin(\theta)^2 d\theta + \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos(\theta)^2 d\theta = \frac{3}{4} \int_0^{\frac{\pi}{6}} \Big(1 - \cos(2\theta)\Big) d\theta + \frac{1}{4} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \Big(1 + \cos(2\theta)\Big) d\theta$$

$$= \frac{3}{4} \left[\theta - \frac{1}{2}\sin(2\theta) \right]_0^{\frac{\pi}{6}} + \frac{1}{4} \left[\theta + \frac{1}{2}\sin(2\theta) \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \frac{3}{4} \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4} \right) + \frac{1}{4} \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} \right) = \frac{5\pi}{24} - \frac{\sqrt{3}}{4}$$

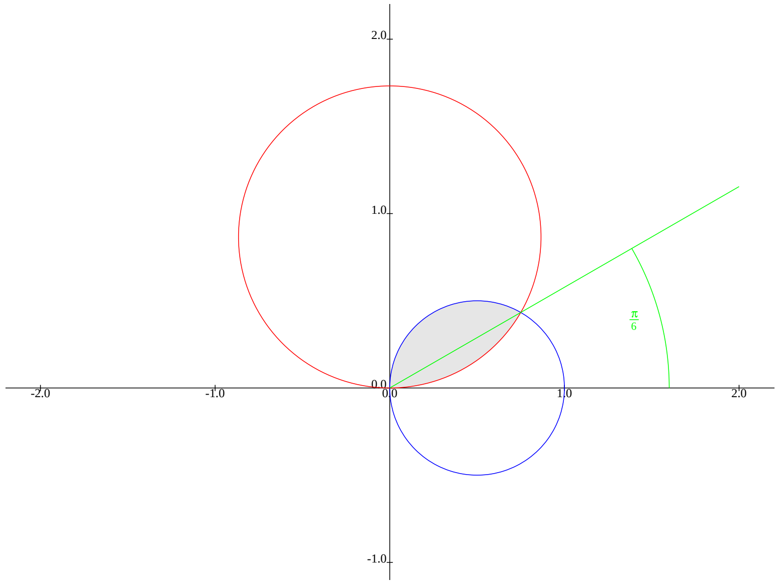

Figure 1: Region for question 22 shown in gray.

23. The two curves meet where $\dfrac{2}{1+x^2} = x^2$, i.e. where $x^4 + x^2 - 2 = 0$. This is a quadratic equation for x^2 giving $(x^2+2)(x^2-1) = 0$, leading to $x = \pm 1$. Since we are only interested in $x \geq 0$, the pieces we are interested in meet at $x = 1$.

The area of A can then be computed as

$$\int_0^1 \Big(\frac{1}{1+x^2} - \frac{1}{2}x^2\Big)dx = \left[\arctan(x) - \frac{1}{6}x^3\right]_0^1 = \frac{\pi}{4} - \frac{1}{6}$$

Alternative answer:

$$\int_0^{\frac{1}{2}} \sqrt{2y}dy + \int_{\frac{1}{2}}^1 \sqrt{\frac{1-y}{y}}dy = \left[\frac{2\sqrt{2}}{3}y^{(\frac{3}{2})}\right]_0^{\frac{1}{2}} + \left[\sqrt{y(1-y)} + \frac{1}{2}\arcsin(2y-1)\right]_{\frac{1}{2}}^1$$
$$= \frac{1}{3} + \left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{\pi}{4} - \frac{1}{6}$$

The volume of the solid obtained by revolving the region A about the y-axis is then

$$2\pi \int_0^1 x\Big(\frac{1}{1+x^2} - \frac{1}{2}x^2\Big)dx = \pi\left[\ln(1+x^2) - \frac{1}{4}x^4\right]_0^1 = \pi\left(\ln(2) - \frac{1}{4}\right)$$

24. Since $y = 2x^{\frac{3}{2}}$, $y' = 3x^{\frac{1}{2}}$ and $1 + (y')^2 = 1 + 9x$.
The desired arclength is then

$$\int_0^7 \sqrt{1+9x}\,dx = \left[\frac{2}{27}(1+9x)^{\frac{3}{2}}\right]_0^7 = \frac{2}{27}\left(64^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) = \frac{2}{27}(512-1) = \frac{1022}{27}$$

One loop of the polar curve is parametrized over $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$. (The other one is over $\pi - \frac{\pi}{4} \leq \theta \leq \pi + \frac{\pi}{4}$).
We find that

$$\frac{dr}{d\theta} = \frac{-\sin(2\theta)}{\sqrt{\cos(2\theta)}}$$

resulting in

$$r^2 + \left(\frac{dr}{d\theta}\right)^2 = \cos(2\theta) + \frac{\sin(2\theta)^2}{\cos(2\theta)} = \frac{1}{\cos(2\theta)}$$

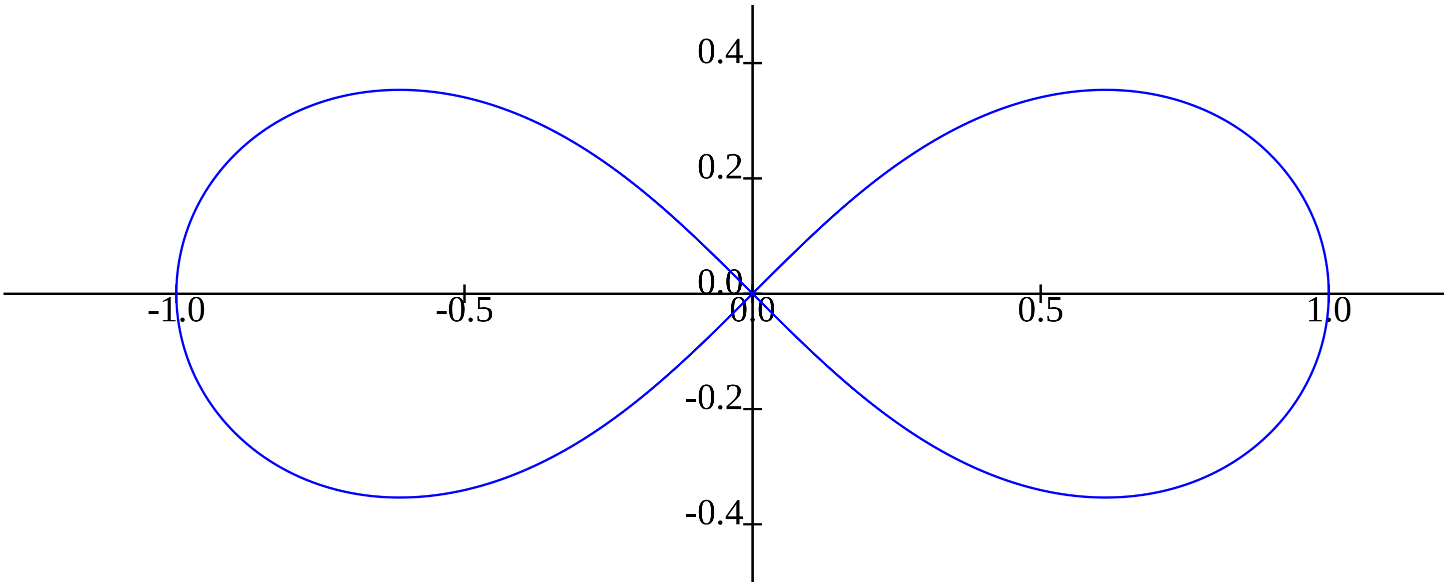

Figure 2: Curve for question 24 second part.

Thus

$$ds = \frac{1}{\sqrt{\cos(2\theta)}} d\theta$$

and the distance from the y-axis is $\sqrt{\cos(2\theta)}\cos(\theta)$. Therefore, the desired surface area is

$$2\pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{\cos(2\theta)}\cos(\theta)\frac{1}{\sqrt{\cos(2\theta)}} d\theta = 2\pi \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos(\theta) d\theta = 2\pi \Big[\sin(\theta)\Big]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = 2\sqrt{2}\pi.$$

25. By the mean value theorem, for $x \geq 0$ we have $\arcsin(x) = \arcsin(x) - \arcsin(0) = \frac{x}{\sqrt{1-\xi^2}} \geq x$ with $0 \leq \xi \leq x$. Therefore $\arcsin\left(\frac{1}{n}\right) \geq \frac{1}{n}$. The series does not converge absolutely by comparison with the harmonic series. The harmonic series diverges because its terms are decreasing and by the integral test.

But $\arcsin\left(\frac{1}{n}\right)$ is decreasing to zero as n increases and therefore the series converges by the alternating series test.

Conclusion: the series is conditionally convergent.

For the second series we use the ratio test. Let $a_n = \frac{(2n)!}{5^n n!(n-1)!}$. Then we have

$$\frac{a_{n+1}}{a_n} = \frac{(2n+2)!5^n n!(n-1)!}{(2n)!5^{n+1}(n+1)!n!} = \frac{(2n+1)(2n+2)}{5n(n+1)} \underset{n\to\infty}{\longrightarrow} \frac{4}{5} < 1$$

The series converges by the ratio test, and since the terms are positive, the series is absolutely convergent.