VERSION **1**

McGill University — Faculty of Science

FINAL EXAMINATION

MATHEMATICS 141 2011 01 CALCULUS 2

EXAMINER: Professor S. W. Drury — DATE: Friday April 15, 2011

ASSOCIATE EXAMINER: Professor N. Sancho — TIME: 9 am. to noon

FAMILY NAME: ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐

GIVEN NAMES: ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐

MR, MISS, MS, MRS, &c.: ☐☐☐☐ STUDENT NUMBER: ☐☐☐☐☐☐☐☐☐

If you expect to graduate in Spring 2011 put an × in this box: ☐

Instructions

1. Fill in the above clearly. Enter your name as it appears on your student card.
2. Do not tear pages from this book; all your writing — even rough work — must be handed in. You may do rough work anywhere in the booklet except in the box below and in the answer boxes.
3. This is a closed book examination. Calculators are not permitted, but regular and translation dictionaries are permitted.
4. This examination consists of two parts, Part A and Part B. READ CAREFULLY THE MORE DETAILLED INSTRUCTIONS AT THE START OF EACH PART.
5. Part A has 20 multiple choice questions each of which is worth 2 points for a total of 40 points. It is recommended that in the first instance, you do not spend more than 4 minutes per question or 75 minutes in total on Part A.
6. Part B has five questions worth a total of 46 points which should be answered on this question paper. The questions in part B are of two types:
   - In BRIEF SOLUTIONS questions, each answer will be marked right or wrong. Within a question, each answer has equal weight.
   - In SHOW ALL YOUR WORK questions a correct answer alone will not be sufficient unless substantiated by your work. Partial marks may be awarded for a partly correct answer.

   In Part B, you are expected to simplify all answers wherever possible.
7. This examination booklet consists of this cover, Pages 1–4 containing questions in Part A, Pages 5–10 containing questions in Part B and Pages 11–14 which are blank continuation pages.
8. A TOTAL OF 86 POINTS ARE AVAILABLE ON THIS EXAMINATION.

PLEASE DO NOT WRITE INSIDE THIS BOX

code	21 /8	22 /8	BS total /16
23 /10	24 /10	25 /10	SA total /30

Part A: MULTIPLE CHOICE QUESTIONS

Each of the following 20 questions is worth 2 points. Half of a point will be subtracted for each wrong answer. The maximum number of points you may earn on these multiple choice questions is 40 points. There is only one correct answer expected for each question. The questions are to be answered on the answer card provided. Be sure to enter on the answer card:

- ◦ Your student number. ◦ Your name.
- ◦ The check code, i.e. the first two letters of your family name.

Fill in the disks below your student number, check code and verify that the marked version number matches the version number on the question paper. If it does not, please notify an invigilator.
It is recommended that in the first instance, you do not spend more than 4 minutes per question or 75 minutes in total on Part A.
Please note:

> The Examination Security Monitor Program detects pairs of students with unusually similar answer patterns on multiple-choice exams. Data generated by this program can be used as admissible evidence, either to initiate or corroborate an investigation or a charge of cheating under Section 16 of the Code of Student Conduct and Disciplinary Procedures.

1. (2 points) Evaluate the Riemann sum for $\int_{-2}^{4} x^2 dx$ which uses three intervals of equal length and evaluation points at the midpoints of these intervals.

(a) 16, (b) 22, (c) 11, (d) $\frac{56}{3}$, (e) 40.

2. (2 points) Find $\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \ln\left(\frac{2n}{n+k}\right)$.

(a) $\ln(2) - \frac{1}{2}$, (b) 1, (c) $\frac{1-\ln(2)}{2}$, (d) $1 - \ln(2)$, (e) $2\ln(2) - 1$.

3. (2 points) Let $F(x) = \int_0^{\sqrt{x}} \sqrt{1+t^4}dt$. Then $F'(1)$ equals

(a) 2, (b) 1, (c) $\sqrt{2}$, (d) $\frac{1}{\sqrt{2}}$, (e) $\frac{1}{2}$.

4. (2 points) Let the function be defined by

$$f(x) = \begin{cases} \sin(x) & \text{if } x < 0, \\ 1 - \cos(x) & \text{if } x \geq 0. \end{cases}$$

Then for $x \geq 0$, $\int_{-\pi}^{x} f(t)dt \; =$

(a) $-2 + x - \sin(x)$, (b) $x - \sin(x)$, (c) $-2 + x + \sin(x)$, (d) $x + \sin(x)$, (e) $x + \cos(x) + \sin(x)$.

5. (2 points) Let $f(x) = \frac{2x}{1+x^2}$. The Integral Mean Value Theorem applied to $\int_1^3 f(t)dt$ asserts the existence of a number a with $1 \leq a \leq 3$ satisfying

(a) $f'(a) = -\frac{1}{4}$, (b) $f(a) = \frac{\ln(5)}{2}$, (c) $f(a) = \ln(5)$, (d) $f'(a) = -\frac{6}{25}$, (e) $f(a) = \frac{4}{5}$.

6. (2 points) $\int_{\frac{\pi}{2}}^{\pi} \frac{\sin(x) + \cos(x)}{\sin(x) - \cos(x)} dx \; =$

(a) $\ln(2)$, (b) 1, (c) 0, (d) $-\ln(2)$, (e) not defined.

7. (2 points) $\int_{\frac{\pi}{2}}^{\pi} \frac{\sin(x) - \cos(x)}{\sin(x) + \cos(x)} dx \; =$

(a) 1, (b) $\ln(2)$, (c) 0, (d) $-\ln(2)$, (e) not defined.

8. (2 points) The area of the region bounded by the parabola $x = y(y - 3)$ and the line $y = x$ is

(a) $\frac{37}{6}$, (b) 9, (c) $\frac{32}{3}$, (d) 8, (e) 5.

9. (2 points) $\int_{-1}^{1} \arctan(x)dx \; =$

(a) $\frac{\pi - \ln(4)}{2}$, (b) $\frac{\pi - \ln(4)}{4}$, (c) π, (d) 0, (e) $\frac{\pi - \ln(2)}{4}$.

10. (2 points) The volume obtained by rotating the region $x > 0$, $0 \leq y < x^{-1}e^{-x}$ about the y-axis is

(a) $\frac{\pi}{2}$, (b) π, (c) 2π, (d) 4π, (e) infinite.

11. (2 points) The volume obtained by rotating the region $x > 0$, $0 \le y < x^{-1}e^{-x}$ about the x-axis is

(a) π, (b) $\frac{\pi}{2}$, (c) 2π, (d) 4π, (e) infinite.

12. (2 points) Using the substitution $x = \cos(t)$ or otherwise, find $\int_{\frac{1}{\sqrt{2}}}^{1} \frac{\sqrt{1-x^2}}{x^2} dx$

(a) $1 - \frac{\pi}{4}$, (b) $\frac{\pi}{4}$, (c) $\frac{\pi - 1}{4}$, (d) $\frac{\pi - 2}{4}$, (e) $\frac{1}{2}$.

13. (2 points) $\int_0^5 \frac{x}{\sqrt{4+x}} dx =$

(a) $\frac{17}{3}$, (b) $\frac{14}{3}$, (c) 4, (d) $\frac{16}{3}$, (e) 5.

14. (2 points) The arclength of the portion of the curve $y = \cosh(x)$ from the point where $x = 0$ to the point where $x = \ln(4)$ is

(a) $\frac{11}{4}$, (b) $\frac{17}{4}$, (c) $\frac{11}{8}$, (d) $\frac{15}{8}$, (e) $\frac{17}{8}$.

15. (2 points) Using the substitution $u = \sqrt{x+1}$ or otherwise, find $\int_3^8 \frac{\sqrt{x+1}}{x} dx$

(a) $2 + \ln(3) - \ln(2)$, (b) $4\sqrt{2} - 2\sqrt{3} - \ln(3) + 3\ln(2)$, (c) $\frac{3\ln(2) - \ln(3)}{2}$,
(d) $10 + \ln(14) - 2\ln(3)$, (e) $2\Big(1 + \arctan(2) - \arctan(3)\Big)$.

16. (2 points) The surface area obtained by rotating the portion of the curve $y = \sqrt{1+x^2}$ from the point where $x = 0$ to the point where $x = 1$ about the x-axis is given by the expression

(a) $\pi \int_0^1 \sqrt{5+4x^2} dx$, (b) $2\pi \int_0^1 \sqrt{1+2x^2} dx$, (c) $2\pi \int_0^1 \sqrt{5+4x^2} dx$,
(d) $2\pi \int_0^1 \sqrt{1+4x^2} dx$, (e) $2\pi \int_0^1 x\sqrt{1+x^2} dx$.

17. (2 points) The tangent to the parametric curve $x = 1 - t^3$, $y = 2t^2$ at the point $(x, y) = (0, 2)$ passes through the point $(x, y) =$

(a) $(-1, 4)$, (b) $(-6, 8)$, (c) $(6, -6)$, (d) $(5, -3)$, (e) $(-4, 6)$.

18. (2 points) Exactly one of the following series is convergent. Which one?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{2^n + n^2}$, (b) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n^2 \ln(n)^2}$, (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+5}$, (d) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2 + 4n}$,
(e) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{3^n + 2^n}$.

19. (2 points) Exactly one of the following series is divergent. Which one?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{2^n + n^2}$, (b) $\displaystyle\sum_{n=3}^{\infty} \frac{n \ln(n) + 1}{n^3}$, (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$, (d) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!}$, (e) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + n^3}{3^n}$.

20. (2 points) Exactly one of the following series is absolutely convergent. Which one?

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 4n}$, (b) $\displaystyle\sum_{n=3}^{\infty} \frac{\ln(n)}{n^{\frac{3}{2}}}$, (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$, (d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$,
(e) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 1}$.

Part B: ANSWER DIRECTLY ON THE QUESTION PAPER

In Part B there are 5 questions each worth a total of 46 points. In Part B, you are expected to simplify all answers wherever possible.

The questions in part B are of two types:

- In BRIEF SOLUTIONS questions worth 8 points each, each answer will be marked right or wrong. Within a question, each answer has equal weight. For these questions start your rough work below the answer box and use the facing page if necessary.
- In SHOW ALL YOUR WORK questions worth 10 points each, a correct answer alone will not be sufficient unless substantiated by your work. Partial marks may be awarded for a partly correct answer. Begin your solution on the page where the question is printed. You may continue a solution *on the facing page*, or on the continuation pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed! Write your final answer in the answer box provided.

21. (8 points) BRIEF SOLUTIONS Given that $x^3 - 2x - 4$ vanishes when $x = 2$, write $\frac{5x}{x^3 - 2x - 4}$ in partial fractions form.

$\frac{5x}{x^3 - 2x - 4} =$ ANSWER ONLY

Find the value of

$$\int_3^\infty \frac{5x}{x^3 - 2x - 4} dx$$

ANSWER ONLY

22. (8 points) BRIEF SOLUTIONS Consider the region R enclosed by both of the polar curves $r = \cos(\theta)$ and $r = \sqrt{3}\sin(\theta)$. Express the area of R as the sum of two or more definite integrals.

Area =

ANSWER ONLY

Now evaluate and simplify the expression you have entered above.

ANSWER ONLY

23. (10 points) SHOW ALL YOUR WORK! Let A be the region of the xy-plane where all of the inequalities $x \geq 0$, $y \leq \dfrac{1}{1+x^2}$ and $2y \geq x^2$ hold. Find the area of the region A.

ANSWER ONLY

Find the volume of the solid obtained by revolving the region A about the y-axis using the method of cylindrical shells.

ANSWER ONLY

You must use the specified method to get credit.

24. (10 points) SHOW ALL YOUR WORK! Find the arclength of the portion of the curve $y = 2x^{\frac{3}{2}}$ between the points corresponding to $x = 0$ and $x = 7$.

ANSWER ONLY

The polar curve $r = \sqrt{\cos(2\theta)}$ consists of two loops. Find the surface area obtained by revolving one of the loops about the y-axis.

ANSWER ONLY

25. (10 points) SHOW ALL YOUR WORK! For each of the following series you should apply one or more tests to determine whether the series is absolutely convergent, conditionally convergent or divergent. All tests used must be named and all statements carefully justified.

(i) $\sum_{n=1}^{\infty}(-1)^n \arcsin\left(\frac{1}{n}\right)$

ANSWER ONLY

(ii) $\sum_{n=1}^{\infty}\frac{(2n)!}{5^n n!(n-1)!}$

ANSWER ONLY

CONTINUATION PAGE FOR PROBLEM NUMBER

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VERSION **2**

McGill University — Faculty of Science

FINAL EXAMINATION

MATHEMATICS 141 2011 01 CALCULUS 2

EXAMINER: Professor S. W. Drury DATE: Friday April 15, 2011
ASSOCIATE EXAMINER: Professor N. Sancho TIME: 9 am. to noon

FAMILY NAME: ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐
GIVEN NAMES: ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐

MR, MISS, MS, MRS, &c.: ☐☐☐☐ STUDENT NUMBER: ☐☐☐☐☐☐☐☐☐

If you expect to graduate in Spring 2011 put an × in this box: ☐

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6. Part B has five questions worth a total of 46 points which should be answered on this question paper. The questions in part B are of two types:
   - In BRIEF SOLUTIONS questions, each answer will be marked right or wrong. Within a question, each answer has equal weight.
   - In SHOW ALL YOUR WORK questions a correct answer alone will not be sufficient unless substantiated by your work. Partial marks may be awarded for a partly correct answer.

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8. A TOTAL OF 86 POINTS ARE AVAILABLE ON THIS EXAMINATION.

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code	21	22	BS total
	/8	/8	/16
23	24	25	SA total
/10	/10	/10	/30

Part A: MULTIPLE CHOICE QUESTIONS

Each of the following 20 questions is worth 2 points. Half of a point will be subtracted for each wrong answer. The maximum number of points you may earn on these multiple choice questions is 40 points. There is only one correct answer expected for each question. The questions are to be answered on the answer card provided. Be sure to enter on the answer card:

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It is recommended that in the first instance, you do not spend more than 4 minutes per question or 75 minutes in total on Part A.
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1. (2 points) Find $\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \ln\left(\frac{2n}{n+k}\right)$.

(a) 1, (b) $2\ln(2)-1$, (c) $\frac{1-\ln(2)}{2}$, (d) $1-\ln(2)$, (e) $\ln(2)-\frac{1}{2}$.

2. (2 points) Let the function be defined by

$$f(x) = \begin{cases} \sin(x) & \text{if } x < 0, \\ 1-\cos(x) & \text{if } x \geq 0. \end{cases}$$

Then for $x \geq 0$, $\int_{-\pi}^{x} f(t)dt =$

(a) $-2+x-\sin(x)$, (b) $-2+x+\sin(x)$, (c) $x-\sin(x)$, (d) $x+\sin(x)$, (e) $x+\cos(x)+\sin(x)$.

3. (2 points) Let $F(x) = \int_0^{\sqrt{x}} \sqrt{1+t^4}dt$. Then $F'(1)$ equals

(a) $\frac{1}{2}$, (b) 2, (c) $\frac{1}{\sqrt{2}}$, (d) 1, (e) $\sqrt{2}$.

4. (2 points) Evaluate the Riemann sum for $\int_{-2}^{4} x^2 dx$ which uses three intervals of equal length and evaluation points at the midpoints of these intervals.

(a) 22, (b) $\frac{56}{3}$, (c) 40, (d) 11, (e) 16.

5. (2 points) Let $f(x) = \frac{2x}{1+x^2}$. The Integral Mean Value Theorem applied to $\int_1^3 f(t)dt$ asserts the existence of a number a with $1 \leq a \leq 3$ satisfying

(a) $f'(a) = -\frac{6}{25}$, (b) $f(a) = \frac{\ln(5)}{2}$, (c) $f(a) = \frac{4}{5}$, (d) $f'(a) = -\frac{1}{4}$, (e) $f(a) = \ln(5)$.

6. (2 points) $\int_{\frac{\pi}{2}}^{\pi} \frac{\sin(x) + \cos(x)}{\sin(x) - \cos(x)} dx =$

(a) $\ln(2)$, (b) 1, (c) 0, (d) $-\ln(2)$, (e) not defined.

7. (2 points) $\int_{\frac{\pi}{2}}^{\pi} \frac{\sin(x) - \cos(x)}{\sin(x) + \cos(x)} dx =$

(a) $\ln(2)$, (b) 0, (c) 1, (d) $-\ln(2)$, (e) not defined.

8. (2 points) $\int_{-1}^{1} \arctan(x) dx =$

(a) $\frac{\pi - \ln(4)}{2}$, (b) $\frac{\pi - \ln(4)}{4}$, (c) $\frac{\pi - \ln(2)}{4}$, (d) π, (e) 0.

9. (2 points) The area of the region bounded by the parabola $x = y(y-3)$ and the line $y = x$ is

(a) $\frac{37}{6}$, (b) 9, (c) 5, (d) $\frac{32}{3}$, (e) 8.

10. (2 points) The volume obtained by rotating the region $x > 0$, $0 \leq y < x^{-1}e^{-x}$ about the y-axis is

(a) 4π, (b) $\frac{\pi}{2}$, (c) π, (d) 2π, (e) infinite.

11. (2 points) The volume obtained by rotating the region $x > 0$, $0 \leq y < x^{-1}e^{-x}$ about the x-axis is

(a) $\frac{\pi}{2}$, (b) 2π, (c) 4π, (d) π, (e) infinite.

12. (2 points) $\int_0^5 \frac{x}{\sqrt{4+x}} dx \ =$

(a) $\frac{16}{3}$, (b) 5, (c) 4, (d) $\frac{17}{3}$, (e) $\frac{14}{3}$.

13. (2 points) The arclength of the portion of the curve $y = \cosh(x)$ from the point where $x = 0$ to the point where $x = \ln(4)$ is

(a) $\frac{11}{4}$, (b) $\frac{15}{8}$, (c) $\frac{11}{8}$, (d) $\frac{17}{4}$, (e) $\frac{17}{8}$.

14. (2 points) The surface area obtained by rotating the portion of the curve $y = \sqrt{1+x^2}$ from the point where $x = 0$ to the point where $x = 1$ about the x-axis is given by the expression

(a) $2\pi \int_0^1 \sqrt{1+4x^2} dx$, (b) $\pi \int_0^1 \sqrt{5+4x^2} dx$, (c) $2\pi \int_0^1 \sqrt{1+2x^2} dx$,
(d) $2\pi \int_0^1 \sqrt{5+4x^2} dx$, (e) $2\pi \int_0^1 x\sqrt{1+x^2} dx$.

15. (2 points) Using the substitution $x = \cos(t)$ or otherwise, find $\int_{\frac{1}{\sqrt{2}}}^1 \frac{\sqrt{1-x^2}}{x^2} dx$

(a) $\frac{\pi - 2}{4}$, (b) $\frac{\pi}{4}$, (c) $1 - \frac{\pi}{4}$, (d) $\frac{1}{2}$, (e) $\frac{\pi - 1}{4}$.

16. (2 points) Using the substitution $u = \sqrt{x+1}$ or otherwise, find $\int_3^8 \frac{\sqrt{x+1}}{x} dx$

(a) $4\sqrt{2} - 2\sqrt{3} - \ln(3) + 3\ln(2)$, (b) $2 + \ln(3) - \ln(2)$, (c) $10 + \ln(14) - 2\ln(3)$,
(d) $\frac{3\ln(2) - \ln(3)}{2}$, (e) $2\Big(1 + \arctan(2) - \arctan(3)\Big)$.

17. (2 points) The tangent to the parametric curve $x = 1 - t^3$, $y = 2t^2$ at the point $(x, y) = (0, 2)$ passes through the point $(x, y) \ =$

(a) $(6, -6)$, (b) $(5, -3)$, (c) $(-1, 4)$, (d) $(-4, 6)$, (e) $(-6, 8)$.

18. (2 points) Exactly one of the following series is divergent. Which one?

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2}$, (b) $\sum_{n=1}^{\infty} \frac{3^n}{n!}$, (c) $\sum_{n=1}^{\infty} \frac{2^n + n^3}{3^n}$, (d) $\sum_{n=3}^{\infty} \frac{n\ln(n) + 1}{n^3}$, (e) $\sum_{n=1}^{\infty} \frac{2^n}{2^n + n^2}$.

19. (2 points) Exactly one of the following series is absolutely convergent. Which one?

(a) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}$, (b) $\sum_{n=1}^{\infty}(-1)^n\frac{n}{n^2+1}$, (c) $\sum_{n=1}^{\infty}(-1)^n\frac{n}{n^2+4n}$, (d) $\sum_{n=3}^{\infty}\frac{\ln(n)}{n^{\frac{3}{2}}}$,

(e) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$.

20. (2 points) Exactly one of the following series is convergent. Which one?

(a) $\sum_{n=3}^{\infty}\frac{1}{n^2\ln(n)^2}$, (b) $\sum_{n=1}^{\infty}\frac{1}{n+5}$, (c) $\sum_{n=1}^{\infty}\frac{2^n}{2^n+n^2}$, (d) $\sum_{n=1}^{\infty}(-1)^n\frac{3^n}{3^n+2^n}$,

(e) $\sum_{n=1}^{\infty}\frac{n}{n^2+4n}$.

Part B: ANSWER DIRECTLY ON THE QUESTION PAPER

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The questions in part B are of two types:

- In BRIEF SOLUTIONS questions worth 8 points each, each answer will be marked right or wrong. Within a question, each answer has equal weight. For these questions start your rough work below the answer box and use the facing page if necessary.
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$\dfrac{5x}{x^3 - 2x - 4} =$ ANSWER ONLY

Find the value of

$$\int_3^\infty \frac{5x}{x^3 - 2x - 4} dx$$

ANSWER ONLY

22. (8 points) BRIEF SOLUTIONS Consider the region R enclosed by both of the polar curves $r = \cos(\theta)$ and $r = \sqrt{3}\sin(\theta)$. Express the area of R as the sum of two or more definite integrals.

Area =

ANSWER ONLY

Now evaluate and simplify the expression you have entered above.

ANSWER ONLY

23. (10 points) SHOW ALL YOUR WORK! Let A be the region of the xy-plane where all of the inequalities $x \geq 0$, $y \leq \dfrac{1}{1+x^2}$ and $2y \geq x^2$ hold. Find the area of the region A.

ANSWER ONLY

Find the volume of the solid obtained by revolving the region A about the y-axis using the method of cylindrical shells.

ANSWER ONLY

You must use the specified method to get credit.

24. (10 points) SHOW ALL YOUR WORK! Find the arclength of the portion of the curve $y = 2x^{\frac{3}{2}}$ between the points corresponding to $x = 0$ and $x = 7$.

ANSWER ONLY

The polar curve $r = \sqrt{\cos(2\theta)}$ consists of two loops. Find the surface area obtained by revolving one of the loops about the y-axis.

ANSWER ONLY

25. (10 points) SHOW ALL YOUR WORK! For each of the following series you should apply one or more tests to determine whether the series is absolutely convergent, conditionally convergent or divergent. All tests used must be named and all statements carefully justified.

(i) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \arcsin\left(\frac{1}{n}\right)$

ANSWER ONLY

(ii) $\displaystyle\sum_{n=1}^{\infty}\frac{(2n)!}{5^n n!(n-1)!}$

ANSWER ONLY

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VERSION **3**

McGill University — Faculty of Science

FINAL EXAMINATION

MATHEMATICS 141 2011 01 CALCULUS 2

EXAMINER: Professor S. W. Drury
ASSOCIATE EXAMINER: Professor N. Sancho

DATE: Friday April 15, 2011
TIME: 9 am. to noon

FAMILY NAME: ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐
GIVEN NAMES: ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐

MR, MISS, MS, MRS, &c.: ☐☐☐☐ STUDENT NUMBER: ☐☐☐☐☐☐☐☐☐

If you expect to graduate in Spring 2011 put an × in this box: ☐

Instructions

1. Fill in the above clearly. Enter your name as it appears on your student card.
2. Do not tear pages from this book; all your writing — even rough work — must be handed in. You may do rough work anywhere in the booklet except in the box below and in the answer boxes.
3. This is a closed book examination. Calculators are not permitted, but regular and translation dictionaries are permitted.
4. This examination consists of two parts, Part A and Part B. READ CAREFULLY THE MORE DETAILLED INSTRUCTIONS AT THE START OF EACH PART.
5. Part A has 20 multiple choice questions each of which is worth 2 points for a total of 40 points. It is recommended that in the first instance, you do not spend more than 4 minutes per question or 75 minutes in total on Part A.
6. Part B has five questions worth a total of 46 points which should be answered on this question paper. The questions in part B are of two types:
   - In BRIEF SOLUTIONS questions, each answer will be marked right or wrong. Within a question, each answer has equal weight.
   - In SHOW ALL YOUR WORK questions a correct answer alone will not be sufficient unless substantiated by your work. Partial marks may be awarded for a partly correct answer.

   In Part B, you are expected to simplify all answers wherever possible.
7. This examination booklet consists of this cover, Pages 1–4 containing questions in Part A, Pages 5–10 containing questions in Part B and Pages 11–14 which are blank continuation pages.
8. A TOTAL OF 86 POINTS ARE AVAILABLE ON THIS EXAMINATION.

PLEASE DO NOT WRITE INSIDE THIS BOX

code	21 /8	22 /8	BS total /16
23 /10	24 /10	25 /10	SA total /30

Part A: MULTIPLE CHOICE QUESTIONS

Each of the following 20 questions is worth 2 points. Half of a point will be subtracted for each wrong answer. The maximum number of points you may earn on these multiple choice questions is 40 points. There is only one correct answer expected for each question. The questions are to be answered on the answer card provided. Be sure to enter on the answer card:

- ∘ Your student number. ∘ Your name.
- ∘ The check code, i.e. the first two letters of your family name.

Fill in the disks below your student number, check code and verify that the marked version number matches the version number on the question paper. If it does not, please notify an invigilator.
It is recommended that in the first instance, you do not spend more than 4 minutes per question or 75 minutes in total on Part A.
Please note:

> The Examination Security Monitor Program detects pairs of students with unusually similar answer patterns on multiple-choice exams. Data generated by this program can be used as admissible evidence, either to initiate or corroborate an investigation or a charge of cheating under Section 16 of the Code of Student Conduct and Disciplinary Procedures.

1. (2 points) Evaluate the Riemann sum for $\int_{-2}^{4} x^2 dx$ which uses three intervals of equal length and evaluation points at the midpoints of these intervals.

(a) 16, (b) 22, (c) 40, (d) $\frac{56}{3}$, (e) 11.

2. (2 points) Let $F(x) = \int_0^{\sqrt{x}} \sqrt{1+t^4}dt$. Then $F'(1)$ equals

(a) $\frac{1}{2}$, (b) $\frac{1}{\sqrt{2}}$, (c) 1, (d) 2, (e) $\sqrt{2}$.

3. (2 points) Find $\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \ln\left(\frac{2n}{n+k}\right)$.

(a) $2\ln(2) - 1$, (b) $1 - \ln(2)$, (c) $\frac{1-\ln(2)}{2}$, (d) $\ln(2) - \frac{1}{2}$, (e) 1.

4. (2 points) Let the function be defined by

$$f(x) = \begin{cases} \sin(x) & \text{if } x < 0, \\ 1 - \cos(x) & \text{if } x \geq 0. \end{cases}$$

Then for $x \geq 0$, $\int_{-\pi}^{x} f(t)dt =$

(a) $-2 + x - \sin(x)$, (b) $x - \sin(x)$, (c) $-2 + x + \sin(x)$, (d) $x + \sin(x)$, (e) $x + \cos(x) + \sin(x)$.

5. (2 points) Let $f(x) = \dfrac{2x}{1+x^2}$. The Integral Mean Value Theorem applied to $\int_{1}^{3} f(t)dt$ asserts the existence of a number a with $1 \leq a \leq 3$ satisfying

(a) $f(a) = \dfrac{\ln(5)}{2}$, (b) $f'(a) = -\dfrac{6}{25}$, (c) $f(a) = \dfrac{4}{5}$, (d) $f'(a) = -\dfrac{1}{4}$, (e) $f(a) = \ln(5)$.

6. (2 points) $\int_{\frac{\pi}{2}}^{\pi} \dfrac{\sin(x) - \cos(x)}{\sin(x) + \cos(x)} dx =$

(a) 1, (b) 0, (c) $-\ln(2)$, (d) $\ln(2)$, (e) not defined.

7. (2 points) $\int_{\frac{\pi}{2}}^{\pi} \dfrac{\sin(x) + \cos(x)}{\sin(x) - \cos(x)} dx =$

(a) 0, (b) 1, (c) $-\ln(2)$, (d) $\ln(2)$, (e) not defined.

8. (2 points) The area of the region bounded by the parabola $x = y(y-3)$ and the line $y = x$ is

(a) $\dfrac{32}{3}$, (b) 5, (c) $\dfrac{37}{6}$, (d) 8, (e) 9.

9. (2 points) $\int_{-1}^{1} \arctan(x)dx =$

(a) $\dfrac{\pi - \ln(4)}{4}$, (b) 0, (c) π, (d) $\dfrac{\pi - \ln(2)}{4}$, (e) $\dfrac{\pi - \ln(4)}{2}$.

10. (2 points) The volume obtained by rotating the region $x > 0$, $0 \leq y < x^{-1}e^{-x}$ about the x-axis is

(a) 4π, (b) π, (c) 2π, (d) $\dfrac{\pi}{2}$, (e) infinite.

11. (2 points) The volume obtained by rotating the region $x > 0$, $0 \le y < x^{-1}e^{-x}$ about the y-axis is

(a) $\frac{\pi}{2}$, (b) 2π, (c) 4π, (d) π, (e) infinite.

12. (2 points) $\int_0^5 \frac{x}{\sqrt{4+x}}dx \;=$

(a) 4, (b) $\frac{17}{3}$, (c) $\frac{14}{3}$, (d) 5, (e) $\frac{16}{3}$.

13. (2 points) Using the substitution $x = \cos(t)$ or otherwise, find $\int_{\frac{1}{\sqrt{2}}}^1 \frac{\sqrt{1-x^2}}{x^2}dx$

(a) $\frac{\pi-1}{4}$, (b) $\frac{\pi}{4}$, (c) $1-\frac{\pi}{4}$, (d) $\frac{\pi-2}{4}$, (e) $\frac{1}{2}$.

14. (2 points) The surface area obtained by rotating the portion of the curve $y = \sqrt{1+x^2}$ from the point where $x = 0$ to the point where $x = 1$ about the x-axis is given by the expression

(a) $2\pi\int_0^1 \sqrt{1+4x^2}dx$, (b) $2\pi\int_0^1 \sqrt{1+2x^2}dx$, (c) $2\pi\int_0^1 x\sqrt{1+x^2}dx$,
(d) $\pi\int_0^1 \sqrt{5+4x^2}dx$, (e) $2\pi\int_0^1 \sqrt{5+4x^2}dx$.

15. (2 points) Using the substitution $u = \sqrt{x+1}$ or otherwise, find $\int_3^8 \frac{\sqrt{x+1}}{x}dx$

(a) $\frac{3\ln(2)-\ln(3)}{2}$, (b) $10+\ln(14)-2\ln(3)$, (c) $4\sqrt{2}-2\sqrt{3}-\ln(3)+3\ln(2)$,
(d) $2\Big(1+\arctan(2)-\arctan(3)\Big)$, (e) $2+\ln(3)-\ln(2)$.

16. (2 points) The arclength of the portion of the curve $y = \cosh(x)$ from the point where $x = 0$ to the point where $x = \ln(4)$ is

(a) $\frac{17}{8}$, (b) $\frac{15}{8}$, (c) $\frac{11}{8}$, (d) $\frac{17}{4}$, (e) $\frac{11}{4}$.

17. (2 points) The tangent to the parametric curve $x = 1-t^3$, $y = 2t^2$ at the point $(x, y) = (0, 2)$ passes through the point $(x, y) \;=$

(a) $(6, -6)$, (b) $(-4, 6)$, (c) $(-6, 8)$, (d) $(5, -3)$, (e) $(-1, 4)$.

18. (2 points) Exactly one of the following series is convergent. Which one?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+5}$, (b) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n^2 \ln(n)^2}$, (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{3^n + 2^n}$, (d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{2^n + n^2}$,
(e) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2 + 4n}$.

19. (2 points) Exactly one of the following series is absolutely convergent. Which one?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$, (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 1}$, (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 4n}$, (d) $\displaystyle\sum_{n=3}^{\infty} \frac{\ln(n)}{n^{\frac{3}{2}}}$,
(e) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$.

20. (2 points) Exactly one of the following series is divergent. Which one?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{2^n + n^2}$, (b) $\displaystyle\sum_{n=3}^{\infty} \frac{n \ln(n) + 1}{n^3}$, (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + n^3}{3^n}$, (d) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!}$, (e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$.

Part B: ANSWER DIRECTLY ON THE QUESTION PAPER

In Part B there are 5 questions each worth a total of 46 points. In Part B, you are expected to simplify all answers wherever possible.

The questions in part B are of two types:

- In BRIEF SOLUTIONS questions worth 8 points each, each answer will be marked right or wrong. Within a question, each answer has equal weight. For these questions start your rough work below the answer box and use the facing page if necessary.
- In SHOW ALL YOUR WORK questions worth 10 points each, a correct answer alone will not be sufficient unless substantiated by your work. Partial marks may be awarded for a partly correct answer. Begin your solution on the page where the question is printed. You may continue a solution *on the facing page*, or on the continuation pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed! Write your final answer in the answer box provided.

21. (8 points) BRIEF SOLUTIONS Given that $x^3 - 2x - 4$ vanishes when $x = 2$, write $\frac{5x}{x^3 - 2x - 4}$ in partial fractions form.

$\frac{5x}{x^3 - 2x - 4} =$ ANSWER ONLY

Find the value of

$$\int_3^{\infty} \frac{5x}{x^3 - 2x - 4} dx$$

ANSWER ONLY

22. (8 points) BRIEF SOLUTIONS Consider the region R enclosed by both of the polar curves $r = \cos(\theta)$ and $r = \sqrt{3}\sin(\theta)$. Express the area of R as the sum of two or more definite integrals.

Area = ANSWER ONLY

Now evaluate and simplify the expression you have entered above.

ANSWER ONLY

23. (10 points) SHOW ALL YOUR WORK! Let A be the region of the xy-plane where all of the inequalities $x \geq 0$, $y \leq \dfrac{1}{1+x^2}$ and $2y \geq x^2$ hold. Find the area of the region A.

ANSWER ONLY

Find the volume of the solid obtained by revolving the region A about the y-axis using the method of cylindrical shells.

ANSWER ONLY

You must use the specified method to get credit.

24. (10 points) SHOW ALL YOUR WORK! Find the arclength of the portion of the curve $y = 2x^{\frac{3}{2}}$ between the points corresponding to $x = 0$ and $x = 7$.

ANSWER ONLY

The polar curve $r = \sqrt{\cos(2\theta)}$ consists of two loops. Find the surface area obtained by revolving one of the loops about the y-axis.

ANSWER ONLY

25. (10 points) SHOW ALL YOUR WORK! For each of the following series you should apply one or more tests to determine whether the series is absolutely convergent, conditionally convergent or divergent. All tests used must be named and all statements carefully justified.

(i) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \arcsin\left(\frac{1}{n}\right)$

ANSWER ONLY

(ii) $\displaystyle\sum_{n=1}^{\infty}\frac{(2n)!}{5^n n!(n-1)!}$

ANSWER ONLY

CONTINUATION PAGE FOR PROBLEM NUMBER

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VERSION 4

McGill University — Faculty of Science

FINAL EXAMINATION

MATHEMATICS 141 2011 01 CALCULUS 2

EXAMINER: Professor S. W. Drury
ASSOCIATE EXAMINER: Professor N. Sancho

DATE: Friday April 15, 2011
TIME: 9 am. to noon

FAMILY NAME: ____________________
GIVEN NAMES: ____________________

MR, MISS, MS, MRS, &c.: ____ STUDENT NUMBER: __________

If you expect to graduate in Spring 2011 put an × in this box: ☐

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4. This examination consists of two parts, Part A and Part B. READ CAREFULLY THE MORE DETAILLED INSTRUCTIONS AT THE START OF EACH PART.
5. Part A has 20 multiple choice questions each of which is worth 2 points for a total of 40 points. It is recommended that in the first instance, you do not spend more than 4 minutes per question or 75 minutes in total on Part A.
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   - In BRIEF SOLUTIONS questions, each answer will be marked right or wrong. Within a question, each answer has equal weight.
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code	21	22	BS total
	/8	/8	/16
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/10	/10	/10	/30

Part A: MULTIPLE CHOICE QUESTIONS

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- ◦ Your student number. ◦ Your name.
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Fill in the disks below your student number, check code and verify that the marked version number matches the version number on the question paper. If it does not, please notify an invigilator.
It is recommended that in the first instance, you do not spend more than 4 minutes per question or 75 minutes in total on Part A.
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1. (2 points) Let $F(x) = \int_0^{\sqrt{x}} \sqrt{1+t^4}dt$. Then $F'(1)$ equals

(a) $\frac{1}{2}$, (b) 2, (c) $\frac{1}{\sqrt{2}}$, (d) 1, (e) $\sqrt{2}$.

2. (2 points) Evaluate the Riemann sum for $\int_{-2}^{4} x^2 dx$ which uses three intervals of equal length and evaluation points at the midpoints of these intervals.

(a) 11, (b) 16, (c) 22, (d) $\frac{56}{3}$, (e) 40.

3. (2 points) Find $\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \ln\left(\frac{2n}{n+k}\right)$.

(a) 1, (b) $2\ln(2)-1$, (c) $\ln(2)-\frac{1}{2}$, (d) $\frac{1-\ln(2)}{2}$, (e) $1-\ln(2)$.

4. (2 points) Let the function be defined by

$$f(x) = \begin{cases} \sin(x) & \text{if } x < 0, \\ 1 - \cos(x) & \text{if } x \geq 0. \end{cases}$$

Then for $x \geq 0$, $\displaystyle\int_{-\pi}^{x} f(t)dt =$

(a) $-2 + x - \sin(x)$, (b) $x - \sin(x)$, (c) $-2 + x + \sin(x)$, (d) $x + \sin(x)$, (e) $x + \cos(x) + \sin(x)$.

5. (2 points) Let $f(x) = \dfrac{2x}{1+x^2}$. The Integral Mean Value Theorem applied to $\displaystyle\int_{1}^{3} f(t)dt$ asserts the existence of a number a with $1 \leq a \leq 3$ satisfying

(a) $f(a) = \ln(5)$, (b) $f(a) = \dfrac{\ln(5)}{2}$, (c) $f'(a) = -\dfrac{6}{25}$, (d) $f(a) = \dfrac{4}{5}$, (e) $f'(a) = -\dfrac{1}{4}$.

6. (2 points) $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \frac{\sin(x) - \cos(x)}{\sin(x) + \cos(x)} dx =$

(a) 1, (b) 0, (c) $-\ln(2)$, (d) $\ln(2)$, (e) not defined.

7. (2 points) $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \frac{\sin(x) + \cos(x)}{\sin(x) - \cos(x)} dx =$

(a) 0, (b) $\ln(2)$, (c) $-\ln(2)$, (d) 1, (e) not defined.

8. (2 points) $\displaystyle\int_{-1}^{1} \arctan(x) dx =$

(a) 0, (b) $\dfrac{\pi - \ln(4)}{4}$, (c) π, (d) $\dfrac{\pi - \ln(4)}{2}$, (e) $\dfrac{\pi - \ln(2)}{4}$.

9. (2 points) The area of the region bounded by the parabola $x = y(y-3)$ and the line $y = x$ is

(a) $\dfrac{32}{3}$, (b) $\dfrac{37}{6}$, (c) 9, (d) 5, (e) 8.

10. (2 points) The volume obtained by rotating the region $x > 0$, $0 \leq y < x^{-1}e^{-x}$ about the x-axis is

(a) π, (b) $\dfrac{\pi}{2}$, (c) 2π, (d) 4π, (e) infinite.

11. (2 points) The volume obtained by rotating the region $x > 0$, $0 \leq y < x^{-1}e^{-x}$ about the y-axis is

(a) 4π, (b) $\frac{\pi}{2}$, (c) π, (d) 2π, (e) infinite.

12. (2 points) $\int_0^5 \frac{x}{\sqrt{4+x}}dx =$

(a) $\frac{17}{3}$, (b) $\frac{14}{3}$, (c) 5, (d) 4, (e) $\frac{16}{3}$.

13. (2 points) Using the substitution $x = \cos(t)$ or otherwise, find $\int_{\frac{1}{\sqrt{2}}}^1 \frac{\sqrt{1-x^2}}{x^2}dx$

(a) $1 - \frac{\pi}{4}$, (b) $\frac{\pi - 1}{4}$, (c) $\frac{\pi - 2}{4}$, (d) $\frac{1}{2}$, (e) $\frac{\pi}{4}$.

14. (2 points) The tangent to the parametric curve $x = 1 - t^3$, $y = 2t^2$ at the point $(x, y) = (0, 2)$ passes through the point $(x, y) =$

(a) $(-4, 6)$, (b) $(6, -6)$, (c) $(5, -3)$, (d) $(-1, 4)$, (e) $(-6, 8)$.

15. (2 points) Using the substitution $u = \sqrt{x+1}$ or otherwise, find $\int_3^8 \frac{\sqrt{x+1}}{x}dx$

(a) $10 + \ln(14) - 2\ln(3)$, (b) $4\sqrt{2} - 2\sqrt{3} - \ln(3) + 3\ln(2)$, (c) $\frac{3\ln(2) - \ln(3)}{2}$,
(d) $2\Big(1 + \arctan(2) - \arctan(3)\Big)$, (e) $2 + \ln(3) - \ln(2)$.

16. (2 points) The arclength of the portion of the curve $y = \cosh(x)$ from the point where $x = 0$ to the point where $x = \ln(4)$ is

(a) $\frac{15}{8}$, (b) $\frac{17}{4}$, (c) $\frac{17}{8}$, (d) $\frac{11}{8}$, (e) $\frac{11}{4}$.

17. (2 points) The surface area obtained by rotating the portion of the curve $y = \sqrt{1 + x^2}$ from the point where $x = 0$ to the point where $x = 1$ about the x-axis is given by the expression

(a) $\pi \int_0^1 \sqrt{5 + 4x^2}dx$, (b) $2\pi \int_0^1 \sqrt{5 + 4x^2}dx$, (c) $2\pi \int_0^1 \sqrt{1 + 4x^2}dx$,
(d) $2\pi \int_0^1 \sqrt{1 + 2x^2}dx$, (e) $2\pi \int_0^1 x\sqrt{1 + x^2}dx$.

18. (2 points) Exactly one of the following series is convergent. Which one?

(a) $\sum_{n=1}^{\infty} \frac{2^n}{2^n + n^2}$, (b) $\sum_{n=3}^{\infty} \frac{1}{n^2 \ln(n)^2}$, (c) $\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{3^n + 2^n}$, (d) $\sum_{n=1}^{\infty} \frac{n}{n^2 + 4n}$,
(e) $\sum_{n=1}^{\infty} \frac{1}{n+5}$.

19. (2 points) Exactly one of the following series is absolutely convergent. Which one?

(a) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 4n}$, (b) $\sum_{n=3}^{\infty} \frac{\ln(n)}{n^{\frac{3}{2}}}$, (c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$, (d) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$,
(e) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$.

20. (2 points) Exactly one of the following series is divergent. Which one?

(a) $\sum_{n=1}^{\infty} \frac{2^n + n^3}{3^n}$, (b) $\sum_{n=3}^{\infty} \frac{n \ln(n) + 1}{n^3}$, (c) $\sum_{n=1}^{\infty} \frac{2^n}{2^n + n^2}$, (d) $\sum_{n=1}^{\infty} \frac{3^n}{n!}$, (e) $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

Part B: ANSWER DIRECTLY ON THE QUESTION PAPER

In Part B there are 5 questions each worth a total of 46 points. In Part B, you are expected to simplify all answers wherever possible.

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$\dfrac{5x}{x^3 - 2x - 4} =$ ANSWER ONLY

Find the value of

$$\int_3^{\infty} \frac{5x}{x^3 - 2x - 4} dx$$

ANSWER ONLY

22. (8 points) BRIEF SOLUTIONS Consider the region R enclosed by both of the polar curves $r = \cos(\theta)$ and $r = \sqrt{3}\sin(\theta)$. Express the area of R as the sum of two or more definite integrals.

Area = ANSWER ONLY

Now evaluate and simplify the expression you have entered above.

ANSWER ONLY

23. (10 points) SHOW ALL YOUR WORK! Let A be the region of the xy-plane where all of the inequalities $x \geq 0$, $y \leq \dfrac{1}{1+x^2}$ and $2y \geq x^2$ hold. Find the area of the region A.

ANSWER ONLY

Find the volume of the solid obtained by revolving the region A about the y-axis using the method of cylindrical shells.

ANSWER ONLY

You must use the specified method to get credit.

24. (10 points) SHOW ALL YOUR WORK! Find the arclength of the portion of the curve $y = 2x^{\frac{3}{2}}$ between the points corresponding to $x = 0$ and $x = 7$.

ANSWER ONLY

The polar curve $r = \sqrt{\cos(2\theta)}$ consists of two loops. Find the surface area obtained by revolving one of the loops about the y-axis.

ANSWER ONLY

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(i) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \arcsin\left(\frac{1}{n}\right)$

ANSWER ONLY

(ii) $\displaystyle\sum_{n=1}^{\infty}\frac{(2n)!}{5^n n!(n-1)!}$

ANSWER ONLY

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