VERSION **1**

McGILL UNIVERSITY
FACULTY OF SCIENCE
FINAL EXAMINATION

MATHEMATICS 141 2007 01 CALCULUS II

EXAMINER: Professor W. G. Brown
ASSOCIATE EXAMINER: Mr. S. Shahabi

DATE: Friday, April 13th, 2007
TIME: 14:00 – 17:00 hours

FAMILY NAME:

GIVEN NAMES:

STUDENT NUMBER:

INSTRUCTIONS

1. Fill in the above clearly.
2. Do not tear pages from this book; all your writing — even rough work — must be handed in. You may do rough work for this paper anywhere in the booklet.
3. Calculators are not permitted. This is a closed book examination. Regular and translation dictionaries are permitted.
4. This examination booklet consists of this cover, Pages 1 through 8 containing questions; and Pages 9, 10, and 11, which are blank. *Your neighbour's version of this test may be different from yours.*
5. There are two kinds of problems on this examination, each clearly marked as to its type.
   - Most of the questions on this paper require that you SHOW ALL YOUR WORK! Their solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed!
   - Some of the questions on this paper require only BRIEF SOLUTIONS ; for these you are expected to write the correct answer in the box provided; you are not asked to show your work, and you should not expect partial marks for solutions that are not correct.

   You are expected to simplify your answers wherever possible.

   You are advised to spend the first few minutes scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)
6. A TOTAL OF 100 MARKS ARE AVAILABLE ON THIS EXAMINATION.

PLEASE DO NOT WRITE INSIDE THIS BOX

1(a) /2	1(b) /2	1(c) /3	1(d) /3	1(e) /4	2(a) /4	2(b) /4	2(c) /4	3(a) /3
3(b) /3	3(c) /3	3(d) /2	3(e) /3	4 /12	5(a) /4	5(b) /5	5(c) /4	6(a) /6
6(b) /6	7(a) /5	7(b) /5	7(c) /3	8 /10				TOTAL /100

1. SHOW ALL YOUR WORK!

   Your answers must be simplified as much as possible.

   (a) [2 MARKS] Evaluate $\int_{-1}^{2} |x|^2 \, dx$.

   (b) [2 MARKS] Evaluate $\int_{1}^{0} \frac{t^4 dt}{\sqrt{t^5+1}}$.

   (c) [3 MARKS] Determine the value of

   $$\frac{1}{n}\left[\left(\frac{0}{n}\right)^3 + \left(\frac{1}{n}\right)^3 + \left(\frac{2}{n}\right)^3 + \ldots + \left(\frac{n-1}{n}\right)^3\right].$$

   (d) [3 MARKS] Suppose it is known that $f'(x) = 4\cosh x$ for all x. Showing all your work, determine the value of $f(1) - f(-1)$, expressed in terms of the values of either exponentials or hyperbolic functions.

   (e) [4 MARKS] Evaluate $\frac{d}{dx}\int_{\frac{1}{2}}^{x^2} e^{t^t} \, dt$ when $x = 1$.

SHOW ALL YOUR WORK!

2. For each of the following series you are expected to apply one or more tests for convergence or divergence to determine whether the series is absolutely convergent, conditionally convergent, or divergent. All tests used must be named, and all statements must be carefully justified.

(a) [4 MARKS] $\displaystyle\sum_{n=1}^{\infty} \frac{(-n-2)^n(n-2)^n}{(2n^2+1)^n}$

(b) [4 MARKS] $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n!}{n^2 2^n}$

(c) [4 MARKS] $\displaystyle\sum_{n=1}^{\infty}(-1)^n \sin\frac{1}{n}$

3. BRIEF SOLUTIONS Express each of the following as a definite integral or a sum, product, or quotient of several definite integrals, simplified as much as possible; you are not expected to evaluate the integrals.

   $\mathcal{R}$ is defined to be the region enclosed by the curves $x + y = 6$ and $y = x^2$; $\mathcal{C}$ is the arc $y = 3^x$ $(-1 \leq x \leq 2)$.

   (a) [3 MARKS] The region $\mathcal{R}$ is rotated about the x-axis. Give an integral or sum of integrals whose value is the volume of the resulting solid.

DEFINITE INTEGRAL(S) ONLY (DO NOT EVALUATE)

   (b) [3 MARKS] The region $\mathcal{R}$ is rotated about the line $x = 5$. Give an integral or sum of integrals whose value is the volume of the resulting solid.

DEFINITE INTEGRAL(S) ONLY (DO NOT EVALUATE)

   (c) [3 MARKS] Express in terms of integrals — which you need not evaluate — the average length that $\mathcal{R}$ cuts off from the vertical lines which it meets.

DEFINITE INTEGRAL(S) ONLY (DO NOT EVALUATE)

   (d) [2 MARKS] Give an integral whose value is the length of $\mathcal{C}$; you need not evaluate the integral.

DEFINITE INTEGRAL(S) ONLY (DO NOT EVALUATE)

   (e) [3 MARKS] Given an integral whose value is the area of the surface generated by rotating $\mathcal{C}$ about the line $y = -1$; you need not evaluate the integral.

DEFINITE INTEGRAL(S) ONLY (DO NOT EVALUATE)

4. SHOW ALL YOUR WORK!

[12 MARKS] Evaluate the indefinite integral

$$\int \frac{x\left(x^2-4\right)(x-2)+4}{\left(x^2-4\right)(x-2)}\,dx\,.$$

5. SHOW ALL YOUR WORK!

Showing all your work, evaluate each of the following:

(a) [4 MARKS] $\int e^{-x} \cdot \cos x \, dx$

(b) [5 MARKS] $\int_{-\frac{1}{2}}^{\frac{5}{2}} \frac{x}{\sqrt{8+2x-x^2}} \, dx$

(c) [4 MARKS] $\int \left(\cos^2 x + \frac{1}{\cos^2 x} \right) \cdot \tan^2 x \, dx$

6. SHOW ALL YOUR WORK!

Consider the arc $\mathcal{C}$ defined by

$$\begin{aligned} x = x(t) &= \cos t + t \sin t \\ y = y(t) &= \sin t - t \cos t \,, \end{aligned}$$

where $0 \leq t \leq \frac{\pi}{2}$.

(a) [6 MARKS] Determine as a function of t the value of $\dfrac{d^2y}{dx^2}$.

(b) [6 MARKS] Determine the area of the surface generated by revolving $\mathcal{C}$ about the y-axis.

7. SHOW ALL YOUR WORK!

(a) [5 MARKS] Showing detailed work, determine whether the following integral is convergent; if it is convergent, determine its value:

$$\int_{\frac{\pi}{2}}^{\pi} \sec x \, dx \, .$$

(b) [5 MARKS] Showing all your work, carefully determine whether the series $\sum_{n=3}^{\infty} \frac{4}{n \ln n}$ is convergent.

(c) [3 MARKS] Showing all your work, determine whether the following sequence converges; if it converges, find its limit:

$$\begin{aligned} a_1 &= 1. \\ a_2 &= 1.23 \\ a_3 &= 1.2345 \\ a_4 &= 1.234545 \\ a_5 &= 1.23454545 \\ a_6 &= 1.2345454545 \end{aligned}$$

etc., where each term after a_2 is obtained from its predecessor by the addition on the right of the decimal digits 45.

8. SHOW ALL YOUR WORK!

[10 MARKS] The polar curves

$$\begin{aligned} r &= 2 + 2\sin\theta \qquad (0 \le \theta \le 2\pi) \\ &\text{and} \\ r &= 6 - 6\sin\theta \qquad (0 \le \theta \le 2\pi) \end{aligned}$$

divide the plane into several regions. Showing all your work, carefully find the area of the region bounded by these curves which contains the point $(r, \theta) = (1, 0)$.

CONTINUATION PAGE FOR PROBLEM NUMBER

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