A six-vector classification of fundamental particles

Joachim Lambek
McGill University, Montreal

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Abstract

An earlier attempt to justify Feynman diagrams for fundamental particles by addition of four-vectors with components $0, 1$ and -1 had ignored the higher generation fermions and the left-hand versus right-hand distinction. The present revised attempt invokes six-vectors instead.

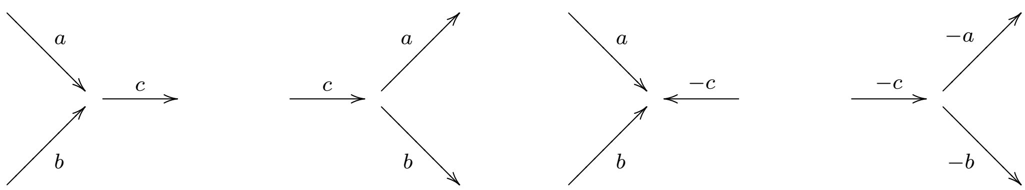
1 Historical introduction

.

The pre-Socratic debate on whether nature is discrete or continuous was resolved in modern physics in favour of the former, at least as far as matter is concerned. Regarding space and time the jury is still out.

In 1979, Harari and Shupe [6,7] proposed that the widely accepted constituents of matter, leptons and quarks, were made up of even smaller constituents. In [8] I pointed out that their proposal could be described mathematically by assigning to each fermion of spin 1/2 a three-vector (a_1, a_2, a_3), where the a_i were all equal to $0, 1$ or -1, -1 being one third of the charge of an electron. I managed to incorporate boson of spin 1 into this description by introducing a fourth number a_0, the *fermion number*, which was 1 for fermions, -1 for anti-fermions and 0 for bosons.

The main purpose of assigning labels to the fundamental particles was (and still is) the justification of the useful *Feynman diagrams*: the labels are attached to arrows meeting at a vertex in such a way that the sum of the incoming arrows is equal to the sum of the outgoing ones. Thus, the equation $a + b = c$ serves to justify the following diagrams:



among others. Note that $-a$ is the label of the anti-particle of the particle labeled a.

2 From four to six

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My earlier description [8] using 4-vectors did not take account of the distinction between left-handed and right-handed fermions, nor of the higher generation ones. Having recently [10] speculated that the quaternionic treatment of special relativity gains in elegance if one allows time and energy to occupy three dimensions, I now realize that the above shortcomings can also be overcome if one replaces the single fermion number a_0 by a three-dimensional "fermion vector" (a'_1, a'_2, a'_3), where the a'_i are also restricted to be $0, 1$ or -1, thus replacing the 4-vector (a_0, a_1, a_2, a_3) by the 6-vector $(a_1, a_2, a_3;\ a'_1, a'_2, a'_3)$.

The old fermion number a_0 can be recaptured from the new fermion vector (a'_1, a'_2, a'_3) by taking the sum of the components module 3:

$$a'_1 + a'_2 + a'_3 \equiv a_0 \pmod 3.$$

What is new is that now

$$(+1, 0, 0),\ (0, +1, 0),\ (0, 0, +1)$$

represent the three generations of left-handed fermions and

$$(0, -1, -1),\ (-1, 0, -1),\ (-1, -1, 0)$$

the three generations of right-handed ones.

Note that

$$0 + -1 + -1 \equiv +1 \pmod 3.$$

The known bosons will be assigned the fermion vector (x, x, x), where $x = 0, 1$ or -1, so that

$$x + x + x \equiv 0 \pmod 3.$$

While one's first choice may be $x = 0$, it is soon realized that the photon ought to be labeled $(0, 0, 0;\ x, x, x)$, with $x = +1$ or -1, to distinguish clockwise and anti-clockwise rotation about the axis of spin. Conventionally, x is called the "helicity".

For weak gauge bosons Z^0, W^+, W^- it turns out that one ought to take $x = -1$, in order to justify the observation that they interact with left-handed fermions only.

For gluons, I see no reason against taking $x = 0$.

Juxtaposing the charge vector and the fermion vector, we obtain the 6-vector

$$(a_1, a_2, a_3;\ a'_1, a'_2, a'_3).$$

Thus we may label the following first generation (red) fermions, leaving the higher generations (and the other colours) to cyclic permutation. For the sake of brevity, we henceforth abbreviate $+1$ and -1 as $+$ and $-$.

$$
\begin{array}{ll}
{}_{L}e^{-} = - - - + 0\ 0 & \text{left-handed electron,} \\
{}_{R}e^{-} = - - - 0 - - & \text{right-handed electron,} \\
{}_{L}\nu_e = 0\ 0\ 0 + 0\ 0 & \text{left-handed electron neutrino,} \\
{}_{R}\nu_e = 0\ 0\ 0\ 0 - - & \text{right-handed electron neutrino (if it exists),} \\
{}_{L}u = 0 + + + 0\ 0 & \text{left-handed up-quark,} \\
{}_{R}u = 0 + + 0 - - & \text{right-handed up-quark,} \\
{}_{L}d = - 0\ 0 + 0\ 0 & \text{left-handed down-quark,} \\
{}_{R}d = - 0\ 0\ 0 - - & \text{right-handed down-quark.}
\end{array}
$$

For antifermions just take the negatives of the above. For example, the antiparticle of the left-handed electron is the *antileft*-handed positron:

$$
{}_{\overline{L}}e^{+} = + + + - 0\ 0\,.
$$

Note that weak gauge bosons interact with left-handed fermions or with antiright-handed antifermions, such as

$$
{}_{\overline{R}}e^{+} = + + + 0\ 0 + +
$$

in the sense that their labels can be added to obtain the label of another fundamental particle.

Until recently it was assumed that right-handed neutrinos do not exist (see Sudbery [13, p.273]), but that antileft antineutrinos do.

Recall that the components of the fermion vector for a boson should add up to 0 modulo 3:

$$
\begin{array}{ll}
\gamma = 0\ 0\ 0 - - - & \text{photon with clockwise spin,} \\
\overline{\gamma} = 0\ 0\ 0 + + + & \text{photon with counterclockwise spin,}
\end{array}
$$

$$
\begin{array}{ll}
Z^0 = 0\ 0\ 0 - - - & \text{neutral weak vector boson} \\
 & \text{(unfortunately with the same label as } \gamma\text{),} \\
W^+ = + + + - - - & \text{positively charged weak vector boson,} \\
W^- = - - - - - - & \text{negatively charged weak vector boson.}
\end{array}
$$

Note that γ and Z^0 are no longer their own antiparticles, nor is W^+ the antiparticle of W^-, as is usually assumed.

There is a remarkable analogy between the charge vector and the fermion vector: colours correspond to generations and the down versus up distinction corresponds to the left versus right one. The difference in sign is due to arbitrary choices made by Benjamin Franklin and the present author.

(I am reminded of the old conundrum: why is it that in a mirror left and right are interchanged but up and down are not? The easiest way to answer this is to point out that east and west are not interchanged but front and back are.)

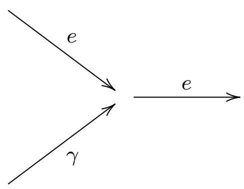
3 Electro-magnetic interaction

.

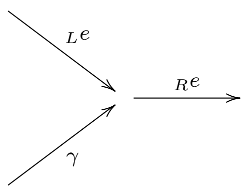
If we adopt the 6-vector

$$\gamma = 0\ 0\ 0 - - -,$$

the old Feynman diagram



must be replaced by

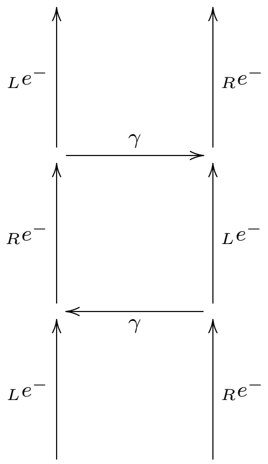


since

$$
\begin{array}{r}
{}_Le^- = - - - + 0\ 0 \\
\gamma = 0\ 0\ 0 - - - \\
\hline
{}_Re^- = - - - \ 0 - -
\end{array}
$$

Adopting Penrose's zigzag picture of the electron, we are thus led to the following illustrations.

3.1 A virtual photon is exchanged between two zigzag electrons.

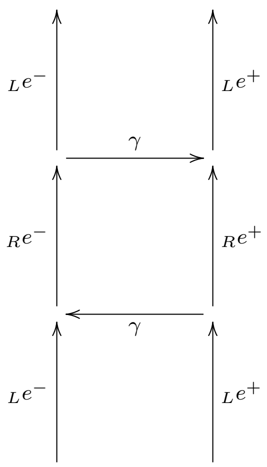


$$
\begin{array}{rcl}
{}_L e^- & = & - - - + 0\ 0 \\
\gamma & = & 0\ 0\ 0 - - - \\
\hline
{}_R e^- & = & - - - 0 - -
\end{array}
$$

Of course, the virtual photon $\xrightarrow{\gamma}$ can be replaced by its antiparticle $\xleftarrow{\bar{\gamma}}$.

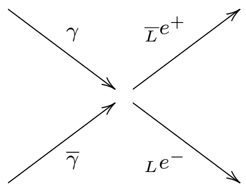
Similarly, a virtual photon can be exchanged between two zigzag fermions or even antifermions. As long as right-handed neutrinos do not exist, we need not consider zigzag neutrinos. (However, Penrose suggests that the zag ${}_R\nu_e$ may be very short.) Instead, neutrinos may oscillate between different generations, as underground observations in Sudbury, Ontario, have recently shown.

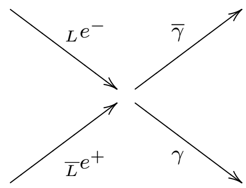
3.2 A virtual photon is exchanged between a zigzag electron and a zigzag positron.



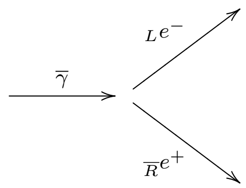
$$
\begin{array}{rcl}
{}_L e^- & = & - - - + 0\ 0 \\
\gamma & = & 0\ 0\ 0 - - - \\
\hline
{}_R e^- & = & - - - 0 - - \\
\\
{}_R e^+ & = & + + + + 0\ 0 \\
\gamma & = & 0\ 0\ 0 - - - \\
\hline
{}_L e^+ & = & + + + 0 - -
\end{array}
$$

3.3 Pair creation and annihilation of an electron and a positron.





may require both γ and $\overline{\gamma}$ as long as we are reluctant to think of a photon traveling backwards in time. However, a single photon will suffice if we allow the handedness of the positron to vary:



since

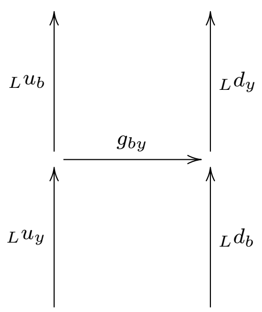
$$
\begin{array}{rcl}
{}_Le^- & = & - - - + 0\ 0 \\
{}_{\overline{R}}e^+ & = & + + + 0 + + \\
\hline
\overline{\gamma} & = & 0\ 0\ 0 + + +
\end{array}
$$

4 Strong and weak interactions

.

Strong interactions are mediated by gluons responsible for colour change. (Traditionally there are also two so-called "diagonal" gluons which do not affect the colour, but I see no reason for introducing them here.) For illustration we have taken blue and yellow quarks, up and down, of the first generation.

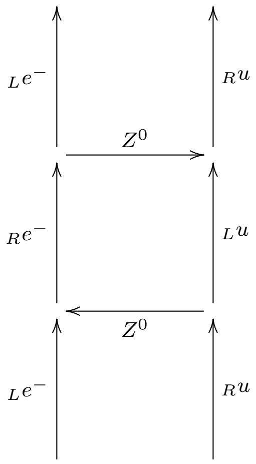
4.1 The force between left-handed quarks due to gluon exchange:



$$
\begin{array}{rcll}
{}_Lu_b & = & + 0 + + 0\ 0 & \text{left blue up} \\
g_{by} & = & 0 + - 0\ 0\ 0 & \text{blue} - \text{yellow} \\
\hline
{}_Lu_y & = & + + 0 + 0\ 0 & \text{left yellow up} \\
\\
{}_Ld_b & = & 0 - 0 + 0\ 0 & \text{left blue down} \\
g_{by} & = & 0 + - 0\ 0\ 0 & \text{blue} - \text{yellow} \\
\hline
{}_Ld_y & = & 0\ 0 - + 0\ 0 & \text{left yellow down}
\end{array}
$$

The scattering of an electron by a Z^0 off an up-quark may be extended to yield the following.

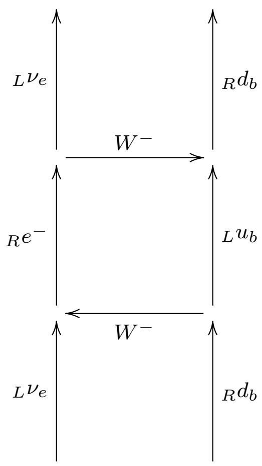
4.2 The exchange of a Z^0 between a zigzag electron and a zigzag up-quark:



$$\begin{array}{rcl@{\qquad}rcl}
_L u & = & 0 + + + 0\ 0 & _L e^- & = & - - - + 0\ 0 \\
Z^0 & = & 0\ 0\ 0 - - - & Z^0 & = & 0\ 0\ 0 - - - \\
\hline
_R u & = & 0 + + 0 - - & _R e^- & = & - - - 0 - -
\end{array}$$

A right-handed electron turns into a left-handed neutrino and a W^-, while a left-handed blue up-quark turns into a right-handed blue down-quark. This may be extended to an exchange between two so-called "doublets".

4.3 The exchange of a W^- between an (e, ν_e) doublet and a (d, u) doublet:



$$\begin{array}{rcl}
_L \nu_e & = & 0\ 0\ 0 + 0\ 0 \\
W^- & = & - - - - - - \\
\hline
_R e^- & = & - - - 0 - -
\end{array}$$

$$\begin{array}{rcl}
_L u_b & = & + 0 + + 0\ 0 \\
W^- & = & - - - - - - \\
\hline
_R d_b & = & 0 - 0\ 0 - -
\end{array}$$

Sometimes we have to be careful in interpreting assertions in the literature. For example, Sudbery [13, p.280] says that $W^+ \to e^+ + \nu_e$. To avoid right-handed neutrinos we can still interpret this as $W^+ \to_R e^+ +_{\overline{L}} \overline{\nu}_e$, since

$$\begin{array}{l}
_R e^+ = + + + 0 - - \\
_{\overline{L}} \overline{\nu}_e = 0\ 0\ 0 - 0\ 0 \\
\hline
W^+ = + + + - - -
\end{array}$$

5 Epilogue

.

As we have seen, the six-vector representation of fundamental particles facilitates the description of handedness and of higher generations. Unfortunately, there are some disadvantages, e.g. the necessity to introduce new anti-particles for Z^0, W^+, W^- and γ. There is also the embarrassing fact that γ and Z^0 have the same label. These disadvantages can be avoided if we allow the fermion vectors of the weak vector bosons to be $(0,0,0)$ instead of $(-1,-1,-1)$ and replace R by L in the diagrams for (4.2) and (4.3), but then we lose our explanation of why these bosons interact with left-handed fermions only.

There are 3^6 different six-vectors made up from the components $0, 1$ and -1. Only a minority of these are labels of known particles, all of spin 1 or 1/2, but there is room to speculate about others whose existence may be predicted. For example, the six different fermion vectors arising from variations of $(-1,+1,0)$ might serve as labels of electrically neutral bosons that help to raise or lower the generation of any fermion, analogous to the colour changing effect of the gluons. In particular, they may account for the observed oscillations of neutrinos between three generations:

$$\begin{array}{rcll} {}_L\nu_e & = & 0\ 0\ 0 + 0\ 0 & \text{first generation} \\ h_{12} & = & 0\ 0\ 0 - + 0 & \text{hypothetical boson} \\ {}_L\nu_\mu & = & 0\ 0\ 0\ 0 + 0 & \text{second generation} \end{array}$$

Here μ is the symbol for a second generation electron, also called a "muon".

Sometimes generation change can be accomplished without introducing new forces. The following example is well-known.

5.1 A second generation electron decays:

$${}_L\mu^- \to {}_Le^- + {}_L\nu_\mu + {}_{\overline{L}}\overline{\nu}_e.$$

Here ${}_{\overline{L}}\overline{\nu}_e$ is the anti-particle of the first generation neutrino ${}_L\nu_e$. Moving the latter to the left side of (5.1), we obtain

$${}_L\mu^- + {}_L\nu_e \to {}_Le^- + {}_L\nu_\mu\ .$$

$$\begin{array}{lcl} {}_L\mu^- = - - - 0 + 0 & & {}_Le^- = - - - + 0\ 0 \\ {}_L\nu_e = 0\ 0\ 0 + 0\ 0 & & {}_L\nu_\mu = 0\ 0\ 0\ 0 + 0 \\ {}_{\overline{R}}\tau^- = - - - + + 0 & & {}_{\overline{R}}\tau^- = - - - + + 0 \end{array}$$

Where ${}_{\overline{R}}\tau^-$ denotes an anti-right "tauon", a tauon being a third generation electron, the existence of which could have been predicted from the existence of the first two generations!

Our new labels also leave room for three generations of *dark* matter, with fermion vectors

$$(- + +),\ (+ - +),\ (+ + -)$$

not admitting addition with (xxx) when $x = \pm 1$ to obtain a fundamental fermion vector, thus describing hypothetical fundamental particles invisible to photons or weak bosons.

Our labels do not account for the graviton and the conjectured Higgs boson. For all I know, these may not be elementary but compounds, as asserted in a recent issue of Scientific American [May 2012, Volume 308, Number 5].

6 Appendix: the quaternion connection

.

In this article we have investigated the classification of fundamental particles and how they fit into Feynman diagrams. For the dynamics of the Standard Model, much more is required, namely Feynman's path-integrals and the calculation of all possible "amplitudes" in which a particle can move. Moreover, many authors have recourse to a more profound mathematical background based on the representation theory of groups and Lie algebras. I have avoided discussing these here, being motivated by matrix representations of the division algebra of quaternions instead.

Quaternions have been used for a century to formulate special relativity. In particular, A.W. Conway and A. Silberstein exploited *biquaternions*, i.e. quaternions with complex components, as early as 1911. Later, the imaginary square root of -1 was replaced by a quaternion unit acting on the right, by C. Lanczos, A.W. Conway and F. Gürsey.

Let me briefly review some pertinent facts. A quaternion has the form

$$a = a_0 + i_1 a_1 + i_2 a_2 + i_3 a_3,$$

where a_0 to a_3 are real numbers and the quaternion *units* i_1 to i_3 satisfy

$$i_1^2 = i_2^2 = i_3^2 = i_1 i_2 i_3 = -1.$$

The resulting algebra admits two *regular* representations by 4×4 matrices.

With any quaternion

$$\psi = \psi_0 + i_1\psi_1 + i_2\psi_2 + i_3\psi_3$$

there is associated the column vector $[\psi]$ consisting of the real numbers ψ_0 to ψ_3. The left and right regular matrix representations L and R are defined by

$$L(a)[\psi] = [a\psi], R(a)[\psi] = [\psi a].$$

It follows that

$$L(ab) = L(a)L(b),\ R(ab) = R(b)R(a),\ L(a)R(b) = R(b)L(a).$$

If $a_0 = 0,\ \mathbf{a} = i_1a_1 + i_2a_2 + i_3a_3$ is called a *vector*. It can be shown that, for any *three-vectors* $\mathbf{a}$ and $\mathbf{a}'$, $L(\mathbf{a}) + R(\mathbf{a}')$ is a 4×4 skew-symmetric matrix and that any such is uniquely of the form

$$A = L(\mathbf{a}) + R(\mathbf{a}').$$

In labelling the fundamental particles, we have restricted the components of $\mathbf{a}$ and $\mathbf{a}'$ to be $0, 1$ or -1, hence A will have integer entries ranging from -2 to $+2$.

If we allow the components of the two vectors to be arbitrary real numbers, we can denote space-time and momentum-energy by

$$X = L(\mathbf{x}) + R(\mathbf{t}),\ P = L(\mathbf{p}) + R(\mathbf{m}),$$

provided we allow $\mathbf{t}$ and $\mathbf{m}$ to occupy three dimensions each, I have suggested [10] that this will render the quaternionic description of special relativity more elegant. To these we add the differential operator D acting on $[\psi]$:

$$D = L(\nabla) - R(\nabla') \quad (\nabla' = \partial/\partial_{x_1} i_1 + \partial/\partial_{x_2} i_2 + \partial/\partial_{x_3} i_3),$$

assuming that $\psi = \psi(X)$ depends on six-dimensional space-time location, and the *conjugates*

$$X^* = L(\mathbf{x}) - R(\mathbf{t}),\ P^* = L(\mathbf{p}) - R(\mathbf{m}),\ D^* = L(\nabla) + R(\nabla').$$

While I did not interpret the two extra dimensions of time and energy, these play a rôle in the promising ideas of Peter Gillan [5], whose theory is based on six-dimensional general relativity.

The so-called *Klein-Gordon* equation

$$DD^*[\psi] = PP^*[\psi] = -\mu^2[\psi] \tag{9.1}$$

may be replaced by two first-order differential equations representing the zig and the zag of the Penrose description

$$D^*[\psi_1] = \mu[\overline{\psi}_2],\ D[\overline{\psi}_2] = -\mu[\psi_1], \tag{9.2}$$

where ψ_1 and ψ_2 are solutions of (9.1) and $\overline{\psi}_2$ is the quaternion conjugate of ψ_2, provided the *rest-mass* μ is assumed to be $\neq 0$.

Now let us make the additional assumption that one of the time dimensions is redundant, say t_3 in a suitable frame of reference, in other words that the action takes place in a five-dimensional subspace of six-dimensional space-time. Then we may obtain a skew-symmetric matrix K such that

$$KK^* = 1,\ KD^* + DK^* = 0,\ KP^* + PK^* = 0,$$

for example, $K = R(i_3)$ in this coordinate system.

Putting

$$[\psi] = [\psi_1] - K[\overline{\psi}_2]$$

we may easily combine the two equations (9.2) into the single *Dirac* equation

$$D^*[\psi] = -\mu K^*[\psi]. \tag{9.3}$$

If we accept the zigzag description by Penrose, there is no need for doing so. However, if

$$K = L(\mathbf{k}) + R(\mathbf{k}'),$$

(9.3) may be rewritten in pure quaternionic form:

$$\vec{\nabla}\psi + \psi\overleftarrow{\nabla}' + \mu(\mathbf{k}\psi - \psi\mathbf{k}') = 0. \tag{9.4}$$

Introducing $\eta = \eta(P)$ by

$$\mu\eta = KP^* = -PK^*,$$

we have $\eta^2 = -1$ and can supply (9.3) with the explicit solution

$$[\psi(X)] = \exp(-\eta(XP^* + PX^*))[\psi(0)].$$

Moreover, putting

$$\eta^T = \mu^{-1} K^* P,$$

we again have $(\eta^T)^2 = -1$ and (9.3) may be rewritten as

$$\eta^T D^*[\psi] = P^*[\psi], \tag{9.6}$$

hence the skew-matrix $\eta^T = \eta^T(P)$ now plays the traditional rôle of the square root of -1 in pre-relativistic quantum mechanics, except now it depends on energy-momentum.

Dirac's equation (9.3) (which also holds trivially when $\mu = 0$) implies

$$[\psi]^T D^*[\psi] = -\mu[\psi[^T K^*[\psi],$$

where the right side is skew-symmetric, hence so must be the left side. Therefore

$$[\psi]^T \overleftrightarrow{D^*}[\psi] = [\psi]^T \overrightarrow{D^*}[\psi] + [\psi]^T \overleftarrow{D^*}[\psi] = 0.$$

If S is any skew-symmetric 4×4 matrix, then so is QSQ^T with the same determinant, provided Q is any 4×4 matrix of determinant 1. Thus $(XX^*)^2 = \det X$ is preserved by the transformation $X \mapsto QXQ^T$. We will call this a *Lorentz transformation* if XX^* itself is invariant, which is easily seen to be equivalent to saying that $X^* \mapsto Q^{-T} X^* Q^{-1}$.

Other physical entities represented by skew symmetric matrices S for which S and S^* transform in the same way as X will be called *basic.* So are P and D and the six-dimensional charge-current density J. It appears that all interesting physical entities A are composed of basic ones and their conjugates, and they transform as

$$A \mapsto Q^u A Q^{-v},$$

where u and v are elements of the monoid $\{0, 1, \sharp\}$, with $Q^0 = I, Q^1 = Q$ and $Q^\sharp = Q^{-T} = (Q^T)^{-1}$. For example $[\psi] \to Q[\psi]$.

If $A = L(\mathbf{a}) + R(\mathbf{a}')$ and $B = L(\mathbf{b}) + R(\mathbf{b}')$ are basic physical entities, their *scalar product*

$$A \odot B = \mathbf{a} \circ \mathbf{b} - \mathbf{b}' \circ \mathbf{a}' = -\frac{1}{2}(AB^* + BA^*)$$

is Lorentz invariant. It extends the Heaviside scalar product $\mathbf{a} \circ \mathbf{b}$ to the six-dimensional $A \odot B$. Note that

$$X \odot X = -s^2, \;\; P \odot P = -\mu^2,$$

where S is called the *interval* and μ the *rest-mass.*

Now let S be any skew-symmetric matrix that is Lorentz invariant and independent of location in space-time, then

$$J_S = [\psi] S [\psi]^T \mapsto Q J_S Q^T$$

is also skew-symmetric and transforms like X. Moreover, one easily calculates the trace of $D^* J_S$:

$$\begin{aligned} -D \odot J_S = & \text{ trace } (D^* J_S) \\ = & \text{ trace } (\overrightarrow{D^*}[\psi] S [\psi]^T) \\ = & \text{ trace } ([\psi]^T \overleftrightarrow{D^*}[\psi] S) \\ = & \ 0, \end{aligned}$$

since the trace of AB equals that of BA.

The equation $D \odot J_S = 0$ resembles Maxwell's equation of continuity and suggests that J_S be regarded as a kind of *charge-current density.* But what is S? It is tempting to choose

$$S = L(\mathbf{s}) + R(\mathbf{s}'),$$

where $(s_1, s_2, s_3; s'_1, s'_2, s'_3)$ is the classifying six-vector of a fundamental particle. If such a particle splits into two at a Feynman vertex, in accordance with the equation $S = U + V$, we may infer that

$$J_S = J_U + J_V.$$

This seems to make sense in view of the probabilistic interpretation of $[\psi]$. However, this does not account for the coupling constant.

References

[1] J. Baez and J. Huerta, the algebra of grand unified theories, Bull. Amer. Math. Soc. **47**(2010), 433-552.

[2] W.H. Davis et al, (eds), Cornelius Lanczos' collected papers with commentaries, Volume iii, North Carolina State University, Raleigh 1998.

[3] R.P. Feynman, QFD, the strange theory of light and matter, Princeton University Press 1985.

[4] M. Gell-Mann, The quark and the jaguar, Freeman, New York 1994.

[5] P. Gillan, How general relativity contains the Standard Model, arχiv:hep-th/011029603 (2010).

[6] H. Harari, The structure of quarks and leptons, Scientific American **248**(1983), 56-68.

[7] H. Harari and M.A. Shupe, Physics Letters 86B (1979), 83-92.

[8] J. Lambek, Four-vector representation of fundamental particles, International Journal of Theoretical Physics **39**(2000), 2253-2358.

[9] ——, In praise of quaternions, Math. Reports, t.a.

[10] ——, Quaternions and three temporal dimensions, Manuscript 2012.

[11] C. Lanczos, Three articles on Dirac's equation, Zeitschrift für Physik **57**(1929), 447-493.

[12] R. Penrose, The road to reality: a complete guide to the laws of the universe, Jonathan Cape, London 2004.

[13] A. Sudbury, Quantum mechanics and the particles of nature, Cambridge Univ. Press 1986.

[14] G. t'Hooft, In search of the ultimate building blocks, Cambridge university Press, 1997.

[15] M. Veltman, Facts and mysteries in elementary particle physics, World Scientific, New Jersey 2003.

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