Champlain College – St.-Lambert

MATH 201-203: Calculus II

Review Questions for Final Exam (2)

Instructor: Dr. Ming Mei

---

1. Find integrals.

   (a) $\displaystyle\int \sin^2 x(1+\cos x)dx$, (b) $\displaystyle\int \sin 2x e^{\cos x} dx$,

   (c) $\displaystyle\int (x^2+1)\ln x dx$, (d) $\displaystyle\int \frac{x^2+1}{x^2-8x+7}dx$.

2. Evaluate each integral and test if it is convergent or divergent.

   (a) $\displaystyle\int_{-\infty}^{0} \frac{2x+1}{(x^2+x+1)^2}dx$, (b) $\displaystyle\int_0^2 \frac{1}{x^2-2x-3}dx$.

3. Let A be a region bounded by $y = \sin x$ and $y = x$ for $x \in [0, \frac{\pi}{4}]$, and V be a solid obtained by rotating A about the x-axis.

   (a) Find the area of A.
   (b) Find the volume of V.

4. Find the solutions to the differential equation:

$$xyy' + \frac{\ln x}{y^2} = 0.$$

5. Test convergence or divergence of the sequences:

   (a) $\displaystyle a_n = \frac{n^2+1}{\sqrt{4n^4-2n^2+1}}$, (b) $\displaystyle a_n = \frac{n\cos n}{n^2+1}$.

6. Test convergence or divergence of the series:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{4n^2+5}$, (b) $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n}+5}{n^2+1}$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{n}{(n^2+1)[\ln(n^2+1)]^2}$, (d) $\displaystyle\sum_{n=0}^{\infty} \frac{5^n+1}{3^{2n}+2^n}$.

7. Find an exact fraction to the decimal number $2.98769876\cdots = 2.\overline{9876}$.

8. Find the interval of convergence of the power series:

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n^3+1}}.$$

9. Find Maclaurin series of the functions:

$$\text{(a) } f(x) = \frac{1}{x+x^3}, \qquad \text{(b) } f(x) = \frac{e^{-x^5}-1}{x^3}.$$

Hints:

1. $\sin^2 x + \cos^2 x = 1$,
2. $\sin 2x = 2\sin x \cos x$
3. $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$.