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ASYMPTOTIC BEHAVIOR OF SOLUTIONS TO THE BIPOLAR HYDRODYNAMIC MODEL OF SEMICONDUCTORS IN BOUNDED DOMAIN

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Abstract. In this paper we present a physically relevant hydrodynamic model for a bipolar semiconductor device considering Ohmic conductor boundary conditions and a non-flat doping profile. For such an Euler-Poisson system, we prove, by means of a technical energy method, that the solutions are unique, exist globally and asymptotically converge to the corresponding stationary solutions. An exponential decay rate is also derived. Moreover we allow that the two pressure functions can be different.

1. **Introduction.** Following the series of studies [15, 16] on the bipolar hydrodynamic system of semiconductors, we consider in this paper a more physical case with non-flat doping profile, different pressure functions, and the Ohmic conductor boundary to the bipolar hydrodynamic system (the coupled system of Euler-Poisson equations)

$$\begin{cases} n_t + j_x = 0, \\ j_t + (\frac{j^2}{n} + p(n))_x = n\phi_x - j, \\ h_t + k_x = 0, \\ k_t + (\frac{k^2}{h} + q(h))_x = -h\phi_x - k, \\ \phi_{xx} = n - h - D(x), \end{cases} \qquad (x,t) \in [0,1] \times \mathbb{R}_+, \tag{1}$$

with the initial data

$$(n, j, h, k)|_{t=0} = (n_0, j_0, h_0, k_0)(x), \quad x \in [0,1] \tag{2}$$

and the Ohmic contact boundary

$$(n, h, \phi)|_{x=0} = (n_l, h_l, 0) \quad \text{and} \quad (n, h, \phi)|_{x=1} = (n_r, h_r, \phi_r), \tag{3}$$

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provided with $n_0(x) > 0$ and $h_0(x) > 0$ for $x \in [0,1]$, and with some given constants $n_l > 0$, $h_l > 0$, $n_r > 0$, $h_r > 0$ and ϕ_r, which satisfy the compatibility conditions $n_0(0) = n_l$, $n_0(1) = n_r$, $h_0(0) = h_l$ and $h_0(1) = h_r$. Here, $n(x,t)$, $h(x,t)$, $j(x,t)$, $k(x,t)$ and $\phi(x,t)$ represent the electron density, the hole density, the current of electrons, the current of holes and the electrostatic potential, respectively. The nonlinear functions $p(s)$ and $q(s)$ denote the pressures of the electrons and the holes, which are usually different and satisfy

$$p, q \in C^3(0, +\infty), \text{ and } s^2 p'(s) \text{ and } s^2 q'(s) \text{ are strictly increasing for } s > 0. \quad (4)$$

$D(x) \in C(0,1)$ is the doping profile standing for the density of impurities in semiconductor devices.

The hydrodynamic models, introduced first by Bløtekjær [3], are used to describe the charged fluid particles such as electrons and holes in semiconductor devices [20, 27] and positively and negatively charged ions in plasmas [20, 34].

For the unipolar semiconductor models (i.e., $h = k = 0$ in (1)), the stationary solutions were first investigated by Degond and Markowich [6], and later by Fang and Ito [7], Gamba [8] and Jerome [19]. The stability of these stationary solutions were then obtained in [11, 17, 18, 21, 24, 31, 32]. Among them, Luo, Natalini and Xin [24] first proved the convergence to stationary solutions for the Cauchy problem in the switch-off case (the current at far fields is zero, or say, the difference of electric field at far fields is zero). Recently, this was ingeniously improved by Huang, Mei, Wang and Yu [17, 18] for the switch-on case. Regarding in the bounded domain, Li, Markowich and Mei [21] obtained the stability of stationary waves for flat doping profile (i.e., the derivative of doping profile $D(x)$ needs to be absolutely small), which was then successfully improved by Guo and Strass [11] and Nishibata and Suzuki [31, 32] independently for non-flat doping profile. For the full system of unipolar hydrodynamic models, the convergence of the smooth solutions to steady-state solutions were archived by Ali, Bini and Rionero [2] and Zhu and Hattori [37] in 1-D switch-off case, and Ali [1] in m-D switch-off case, and further improved by Mei and Wang [29] in m-D switch-on case. For the other interesting studies on the entropy weak solutions and limit of relaxation times, we refer to [4, 5, 10, 13, 22, 26, 33, 36] and the references therein.

For the bipolar semiconductor models, the related study is very limited so far, due to the complexity of structure of the system and other technical difficulty points. In 1996, Natalini [30] first proved the existence of entropy weak solutions of the bipolar hydrodynamic system and further showed the convergence of the entropy solutions to the solutions of the corresponding classical drift-diffusion equations, which was then extended to the bounded domain case by Hsiao and Zhang [12, 13]. In the switch-off case, when the doping profile is completely flat (i.e., $D(x) = 0$), and the two pressure-density functions are exactly identical, i.e., $p(s) = q(s)$ for $s > 0$, Gasser, Hsiao and Li [9] showed that the smooth solutions of the Cauchy problem to the bipolar hydrodynamic model converge to the corresponding diffusion waves, i.e. the self-similar solutions to the corresponding porous media equations. A similar result for weak entropy solutions was also obtained by Huang and Li in [14]. When the system is in the switch-on case, there exist some L^2-gaps between the original solutions and their corresponding diffusion waves, by constructing the correction functions to delete those L^2-gaps, Huang, Mei and Wang [15] successfully solved the L^∞-stability of the diffusion waves even in multidimensional space. The stability of diffusion waves for the bipolar hydrodynamic system with boundary effect in half-space was further proved by Huang, Mei, Wang and Yang [16] with a different method. As we know, the more physically relevant, but challenging case for the bipolar hydrodynamic model, never treated so far, is the system with non-flat doping profile, two distinct pressure-density functions, and the physical Ohmic contact boundary conditions.

When the doping profile is completely flat: $D(x) = 0$, and two pressure functions are identical: $p(s) = q(s)$, the authors [9, 15, 16], by the variable scaling method [23, 25, 28], observed that the diffusion waves $(\bar{n}, \bar{j}, \bar{n}, \bar{j}, 0)(x/\sqrt{1+t})$ are the kind of asymptotic profiles for the original solutions $(n, j, h, k, E)(x, t)$ to (1), where $(\bar{n}, \bar{j})(x/\sqrt{1+t})$ are the solutions to the following nonlinear diffusion (porous media) equations

$$\begin{cases} \bar{n}_t + \bar{j}_x = 0, \\ p(\bar{n})_x = -\bar{j}, \end{cases} \quad \text{or equivalently,} \quad \begin{cases} \bar{n}_t - p(\bar{n})_{xx} = 0, \\ p(\bar{n})_x = -\bar{j}. \end{cases}$$

In those papers, $(n, j)(x, t)$ and $(h, k)(x, t)$ behave very similarly, and share the same asymptotic profiles $(\bar{n}, \bar{j})(x/\sqrt{1+t})$. However, when the doping profile is non-zero: $D(x) \neq 0$, and two pressure functions are different: $p(s) \neq q(s)$, one can observe from $(1)_5$ that the behavior of $n(x, t)$ and $h(x, t)$ are totally different, and $E(x, t) \not\to 0$ as $t \to \infty$, so it is clear that the diffusion waves $(\bar{n}, \bar{j}, \bar{n}, \bar{j}, 0)(x/\sqrt{1+t})$ are no long the asymptotic profiles of the original solutions $(n, j, h, k, E)(x, t)$ for the impure bipolar semiconductor device with two different pressures. So, what will the asymptotic profiles be for the original solutions in this case? Inspired by the study on the unipolar case [11, 17, 18, 21, 24, 31, 32], we expect also the corresponding stationary solutions as the asymptotic profiles. In fact, let us scale the time variable with arbitrary small number $\varepsilon > 0$ as

$$x \to x, \qquad t \to t/\varepsilon, \qquad (n, j, h, k, \phi) \to (\bar{n}, \bar{j}, \bar{h}, \bar{k}, \bar{\phi}),$$

then from (1) we get

$$\begin{cases} \varepsilon \bar{n}_t + \bar{j}_x = 0, \\ \varepsilon \bar{j}_t + (\frac{\bar{j}^2}{\bar{n}} + p(\bar{n}))_x = \bar{n}\bar{\phi}_x - \bar{j}, \\ \varepsilon \bar{h}_t + \bar{k}_x = 0, \\ \varepsilon \bar{k}_t + (\frac{\bar{k}^2}{\bar{h}} + q(\bar{h}))_x = -\bar{h}\bar{\phi}_x - \bar{k}, \\ \bar{\phi}_{xx} = \bar{n} - \bar{h} - D(x). \end{cases}$$

If we neglect the small terms with ε, we then obtain the asymptotic profiles to (1) as follows

$$\begin{cases} \bar{j}_x = 0, \\ (\frac{\bar{j}^2}{\bar{n}} + p(\bar{n}))_x = \bar{n}\bar{\phi}_x - \bar{j}, \\ \bar{k}_x = 0, \\ (\frac{\bar{k}^2}{\bar{h}} + q(\bar{h}))_x = -\bar{h}\bar{\phi}_x - \bar{k}, \\ \bar{\phi}_{xx} = \bar{n} - \bar{h} - D(x) \end{cases} \tag{5}$$

with the Ohmic contact conditions

$$(\bar{n}, \bar{h}, \bar{\phi})|_{x=0} = (n_l, h_l, 0) \quad \text{and} \quad (\bar{n}, \bar{h}, \bar{\phi})|_{x=1} = (n_r, h_r, \phi_r), \tag{6}$$

which are the steady-state solutions, the so-called stationary waves.

In this paper, in order to prove that the IBVP solutions of (1)-(3) exponentially converge to the stationary solutions of (5) and (6) for the case with the non-flat doping profile and two distinct pressure functions, we will adopt the elementary energy method with some technical skill. In fact our approach differs from the one used for the unipolar semiconductor model in [17, 18, 21, 24, 31, 32, 37], where the major working equation after perturbation is a single damped wave equation, which makes us easily establish the *a priori* energy estimates. But it also differs from the bipolar model studied in [9, 15, 16] for $D(x) = 0$ and $p(s) = q(s)$, which ensures the asymptotic profiles for both (n, j) and (h, k) to be exactly the same, and leads that the major working equation after perturbation can be a single Klein-Gordon equation, so then the *a priori* energy estimates can be also easily established. In this paper the major working equations after perturbation will be a system of two damped wave equations and one Poisson equation with the linear parts all involving three unknown variables, which makes that the energy estimates cannot be directly obtained in the usual way. By a heuristic observation, we technically modify the two damped wave equations and suitably combine them with the Poisson equation to delete all negative terms, so that we can establish the *a priori* energy estimates (see Lemma 3.4 later). This is our key point to prove the exponential convergence.

The paper is organized as follows. In section 2, we are going to state the main theorems: the first establishes the existence and uniqueness of the stationary solutions of (5) and (6), and the other states the existence and uniqueness of the original IBVP solutions of (1)-(3) as well as the exponential convergence to the stationary solutions. Section 3 is devoted to the proof of the convergence theorem by a technical energy method.

In what follows, C always denotes a generic positive constant, and C_i for $i = 1, 2, \cdots$ stand for some specific constants. $L^2(0, 1)$ is the space of square-integrable real-valued functions defined on $[0, 1]$, and its norm is denoted by $\|\cdot\|$. $H^l(0, 1)$ is the usual Sobolev space with the norm $\|\cdot\|_l$, so in particular $\|\cdot\|_0 = \|\cdot\|$. For simplicity, we also denote $\|(f, g, h)\|^2 = \|f\|^2 + \|g\|^2 + \|h\|^2$ and $\|(f, g, h)\|_l^2 = \|f\|_l^2 + \|g\|_l^2 + \|h\|_l^2$. Let $T > 0$ and $\mathbf{B}$ be a Banach space. $C^0([0, T]; \mathbf{B})$ is the space of $\mathbf{B}$-valued continuous functions on $[0, T]$, and $L^2([0, T]; \mathbf{B})$ is the space of $\mathbf{B}$-valued L^2-functions on $[0, T]$. The other spaces of $\mathbf{B}$-valued functions on $[0, \infty)$ can be defined similarly.

2. Main results.

In this section, we state the existence and uniqueness of the stationary solutions of (5) and (6) and the convergence of the IBVP solutions (1)-(3) to the stationary solutions.

Clearly, dividing $(5)_2$ by n and $(5)_4$ by h, and integrating the resultant equations over $[0, 1]$ with respect to x, we get, from the equation $(5)_5$ and the boundary condition (6),

$$\bar{\phi}(x) = \int_0^x \int_0^y (\bar{n} - \bar{h} - D)(z)dzdy + \Big(\phi_r - \int_0^1 \int_0^y (\bar{n} - \bar{h} - D)(z)dzdy\Big)x. \quad (7)$$

Dividing the second and the fourth equation of (5) by $\bar{n}$ and $\bar{h}$ and differentiating them with respect to x, respectively, we have

$$\begin{cases} \bar{j} = \mathcal{C}_1, \\ \Big(\Big(\frac{p'(\bar{n})}{\bar{n}} - \frac{\bar{j}^2}{\bar{n}^3}\Big)\bar{n}_x\Big)_x = \bar{n} - \bar{h} - D(x) + \bar{j}\frac{\bar{n}_x}{\bar{n}^2}, \\ \bar{k} = \mathcal{C}_2, \\ \Big(\Big(\frac{q'(\bar{h})}{\bar{h}} - \frac{\bar{k}^2}{\bar{h}^3}\Big)\bar{h}_x\Big)_x = \bar{h} - \bar{n} + D(x) + \bar{k}\frac{\bar{h}_x}{\bar{h}^2}, \end{cases} \quad (8)$$

where $\mathcal{C}_1$ and $\mathcal{C}_2$ are some constants.

To keep the ellipticity of the system (8), we need

$$\frac{1}{\bar{n}}p'(\bar{n}) - \frac{\bar{j}^2}{\bar{n}^3} > 0 \quad \Leftrightarrow \quad \bar{n}^2 p'(\bar{n}) > \bar{j}^2, \tag{9}$$

$$\frac{1}{\bar{h}}q'(\bar{h}) - \frac{\bar{k}^2}{\bar{h}^3} > 0 \quad \Leftrightarrow \quad \bar{h}^2 q'(\bar{h}) > \bar{k}^2, \tag{10}$$

which imply that the velocities of electrons and holes must satisfy

$$|(\bar{u},\bar{v})| := \left|\left(\frac{\bar{j}}{\bar{n}}, \frac{\bar{k}}{\bar{h}}\right)\right| < \left|\left(\sqrt{p'(\bar{n})}, \sqrt{q'(\bar{h})}\ \right)\right| =: c(\bar{n},\bar{h}) \text{ (the speed of sound)},$$

namely, the system describes a fully subsonic flow. Since both $s^2p'(s)$ and $s^2q'(s)$ are increasing for $s > 0$ (see (4)), we can conclude that there is a minimum value $(\bar{n}_m, \bar{h}_m) > 0$ such that (9) and (10) hold for $(\bar{n}, \bar{h}) > (\bar{n}_m, \bar{h}_m)$. Thus, initially we need to assume

$$\min_{x\in[0,1]} \Big(n_0^2(x)p'(n_0(x)) - j_0^2(x)\Big) > 0, \quad \min_{x\in[0,1]} \Big(h_0^2(x)q'(h_0(x)) - k_0^2(x)\Big) > 0. \tag{11}$$

Notice that, when $p(s) = q(s) = s$, $n_l = n_r$ and $h_l = h_r$, Tsuge [35] proved the existence and uniqueness of the stationary solutions for non-flat doping profile. In a similar way as in [31, 32, 35], we can also prove it as follows for $p(s) \neq q(s)$, $n_l \neq n_r$ and $h_l \neq h_r$. The detail is omitted.

Theorem 2.1 (Existence of stationary solutions)**.** *Let $\delta := |n_l - n_r| + |h_l - h_r| + |\phi_r|$ be small enough. Then there exist the unique classical stationary solutions $(\bar{n}, \bar{j}, \bar{h}, \bar{k}, \bar{\phi})(x)$ to the system (5) with the boundary conditions (6), such that*

$$p'(\bar{n}) - \frac{\bar{j}^2}{\bar{n}^2} > 0, \quad x \in [0,1], \tag{12}$$

$$q'(\bar{h}) - \frac{\bar{k}^2}{\bar{h}^2} > 0, \quad x \in [0,1], \tag{13}$$

$$C_{1m} \le \bar{n}(x) \le C_{1M}, \quad x \in [0,1], \tag{14}$$

$$C_{2m} \le \bar{h}(x) \le C_{2M}, \quad x \in [0,1], \tag{15}$$

$$\sup_{x\in[0,1]} \Big\{ \sum_{i=1}^{2} [|\partial_x^i \bar{n}| + |\partial_x^i \bar{h}|] + \sum_{i=0}^{2} [|\partial_x^i \bar{j}| + |\partial_x^i \bar{k}| + |\partial_x^i \bar{\phi}|] \Big\} \le C\delta, \tag{16}$$

where

$$C_{1m} = \inf_{x\in[0,1]} \{n_l,\ n_r,\ |D(x)|\}, \qquad C_{1M} = \sup_{x\in[0,1]} \{n_l,\ n_r,\ |D(x)|\}, \tag{17}$$

$$C_{2m} = \inf_{x\in[0,1]} \{h_l,\ h_r,\ |D(x)|\}, \qquad C_{2M} = \sup_{x\in[0,1]} \{h_l,\ h_r,\ |D(x)|\}. \tag{18}$$

Now we state our main theorem, the convergence result of the original IBVP solutions of (1)-(3) to the stationary solutions of (5) and (6).

Let

$$\begin{cases} N_0(x) := n_0(x) - \bar{n}(x), \\ J_0(x) := j_0(x) - \bar{j}(x), \\ H_0(x) := h_0(x) - \bar{h}(x), \\ K_0(x) := k_0(x) - \bar{k}(x). \end{cases} \tag{19}$$

Theorem 2.2 (Convergence to stationary solutions)**.** *Let $(N_0, J_0, H_0, K_0) \in H^2(0,1)$. Then there exists a number $\varepsilon_0 > 0$ such that, when $\|(N_0, J_0, H_0, K_0)\|_2 + \delta < \varepsilon_0$, then the solutions $(n, j, h, k, \phi)(x,t)$ of (1)-(3) uniquely and globally exist and converge to the stationary solutions $(\bar{n}, \bar{j}, \bar{h}, \bar{k}, \bar{\phi})(x)$ of (5) and (6) in the form of*

$$\max_{x\in[0,1]} |(n, j, h, k, \phi)(x,t) - (\bar{n}, \bar{j}, \bar{h}, \bar{k}, \bar{\phi})(x)| \le Ce^{-\mu t}, \tag{20}$$

where $\mu > 0$ is a constant.

3. **Proof of convergence.**

This section is devoted to the proof of Theorem 2.2. Let

$$\begin{cases} N(x,t) := n(x,t) - \bar{n}(x), \\ J(x,t) := j(x,t) - \bar{j}(x), \\ H(x,t) := h(x,t) - \bar{h}(x), \\ K(x,t) := k(x,t) - \bar{k}(x), \\ \Phi(x,t) := \phi(x,t) - \bar{\phi}(x). \end{cases} \tag{21}$$

From (1)-(3) and (5)-(6), we have

$$\begin{cases} N_t + J_x = 0, \\ J_t + \left(\frac{(\bar{j}+J)^2}{N+\bar{n}} - \frac{\bar{j}^2}{\bar{n}} + p(N+\bar{n}) - p(\bar{n}) \right)_x = (N+\bar{n})\Phi_x + \bar{\phi}_x N - J, \\ H_t + K_x = 0, \\ K_t + \left(\frac{(K+\bar{k})^2}{H+\bar{h}} - \frac{\bar{k}^2}{\bar{h}} + q(H+\bar{h}) - q(\bar{h}) \right)_x = -(H+\bar{h})\Phi_x - \bar{\phi}_x H - K, \\ \Phi_{xx} = N - H, \end{cases} \tag{22}$$

with the initial data

$$(N, J, H, K)|_{t=0} = (N_0, J_0, H_0, K_0)(x), \quad x \in [0,1] \tag{23}$$

and the null Dirichlet boundary conditions

$$(N, H, \Phi)|_{x=0} = (0,0,0) \quad \text{and} \quad (N, H, \Phi)|_{x=1} = (0,0,0). \tag{24}$$

First of all, for $T > 0$, let us define the solution space

$$\begin{aligned} X(0,T) := \quad \{ \quad & (N, J, H, K, \Phi) \mid N,\ H \in C^0(0,T; H^2(0,1)) \cap L^2(0,T; H^2(0,1)), \\ & N_t,\ H_t \in C^0(0,T; H^1(0,1)) \cap L^2(0,T; H^1(0,1)) \\ & J,\ K \in C^0(0,T; H^2(0,1)), \quad J_t,\ K_t \in C^0(0,T; H^1(0,1)) \\ & \Phi \in C^0(0,T; H^3(0,1), \Phi_{xx} \in L^2(0,T; H^3(0,1))\}, \end{aligned}$$

and

$$M(T) := \sup_{t\in[0,T]} \{\|(N, J, H, K)(t)\|_2 + \|(N_t, J_t, H_t, K_t)(t)\|_1 + \|\Phi(t)\|_3\}.$$

We now state the following convergence theorem.

Theorem 3.1. *Let $(N_0, J_0, H_0, K_0) \in H^2(0,1)$. Then there exists a number $\varepsilon_0 > 0$ such that, when $\|(N_0, J_0, H_0, K_0)\|_2 + \delta < \varepsilon_0$, then the solutions of (22)-(24) uniquely and globally exist, with*

$$(N, J, H, K, \Phi) \in X(0, \infty),$$

and satisfy

$$\begin{aligned}&\|(N,J,H,K)(t)\|_2+\|(N_t,J_t,H_t,K_t)(t)\|_1+\|\Phi(t)\|_3\\&+\int_0^t e^{-\mu(t-s)}[\|(N,H)(s)\|_2^2+\|(N_t,H_t,\Phi_{xx})(s)\|_1^2]ds\\&\le Ce^{-\mu t}.\end{aligned} \tag{25}$$

Clearly, Theorem 2.2 can be immediately obtained once Theorem 3.1 is proved.To prove it, we will apply the energy method with the continuation argument based on the local existence result and the *a priori* estimates. The local existence can be obtained by the standard energy estimates with the iteration skill, so we omit it in detail. To establish the *a priori* estimates is crucial, and this will be our aim in the rest of the section.

Differentiating $(22)_2$ and $(22)_4$ in x and substituting $J_x=-N_t$, $K_x=-H_t$ and $\Phi_{xx}=N-H$, respectively, then we have

$$N_{tt}+N_t-F_1(N,J)_{xx}+\bar n(N-H)=F_2(N,H,\Phi_x), \tag{26}$$

and

$$H_{tt}+H_t-G_1(H,K)_{xx}+\bar h(H-N)=G_2(N,H,\Phi_x), \tag{27}$$

where

$$F_1:=\frac{(\bar j+J)^2}{N+\bar n}-\frac{\bar j^2}{\bar n}+p(N+\bar n)-p(\bar n), \tag{28}$$

$$F_2:=-(N_x+\bar n_x)\Phi_x-N(N-H)-\bar\phi_xN_x-N\bar\phi_{xx}, \tag{29}$$

$$G_1:=\frac{(K+\bar k)^2}{H+\bar h}-\frac{\bar k^2}{\bar h}+q(H+\bar h)-q(\bar h), \tag{30}$$

$$G_2:=(H_x+\bar h_x)\Phi_x+H(N-H)+H_x\bar\phi_x+H\bar\phi_{xx}. \tag{31}$$

Lemma 3.2. *Under the hypotheses of Theorem 3.1, it holds*

$$\|(N,H)(t)\|\le 2\|(N_x,H_x)(t)\|, \tag{32}$$

$$\|\Phi(t)\|\le 2\|\Phi_x(t)\|\le 4\|\Phi_{xx}(t)\|\le C\|(N,H)(t)\|. \tag{33}$$

Proof. Since $N|_{x=0}=N_{x=1}=0$, $H|_{x=0}=H_{x=1}=0$ and $\Phi|_{x=0}=\Phi|_{x=1}=0$, by the Poincaré inequality we immediately obtain

$$\|N(t)\|\le 2\|N_x(t)\|,\quad \|H(t)\|\le 2\|H_x(t)\|,\quad \|\Phi(t)\|\le 2\|\Phi_x(t)\|.$$

For the second inequality of (33), although the boundary of Φ_x may not be zero, we can still prove it from the equation $(22)_5$. In fact, multiplying $(22)_5$ by Φ and integrating it by parts with respect to x over $[0,1]$, and applying Hölder inequality, we have

$$\begin{aligned}\int_0^1\Phi_x^2dx&=-\int_0^1(N-H)\Phi dx\le\Big(\int_0^1(N-H)^2dx\Big)^{1/2}\Big(\int_0^1\Phi^2dx\Big)^{1/2}\\&=\Big(\int_0^1\Phi_{xx}^2dx\Big)^{1/2}\Big(\int_0^1\Phi^2dx\Big)^{1/2}\le 2\Big(\int_0^1\Phi_{xx}^2dx\Big)^{1/2}\Big(\int_0^1\Phi_x^2dx\Big)^{1/2},\end{aligned}$$

where, in the last step we used $\|\Phi(t)\|\le 2\|\Phi_x(t)\|$, which then gives

$$\|\Phi_x(t)\|\le 2\|\Phi_{xx}(t)\|.$$

From the equation $\Phi_{xx}=N-H$, the third inequality of (33) is obvious. $\square$

Lemma 3.3. *Under the hypotheses of Theorem 3.1, it holds*

$$\|J(t)\|^2 \le Ce^{-2\mu_0 t}\|J_0\|^2 + C\|(N, N_t, H, H_t)(t)\|^2, \tag{34}$$
$$\|K(t)\|^2 \le Ce^{-2\mu_0 t}\|H_0\|^2 + C\|(N, N_t, H, H_t)(t)\|^2, \tag{35}$$
$$\|J_t(t)\|^2 \le Ce^{-2\mu_0 t}\|J_0\|^2 + C\|(N, N_x, N_t, H, H_x, H_t)(t)\|^2, \tag{36}$$
$$\|K_t(t)\|^2 \le Ce^{-2\mu_0 t}\|H_0\|^2 + C\|(N, N_x, N_t, H, H_x, H_t)(t)\|^2, \tag{37}$$

for some constant $\mu_0 > 0$.

Proof. Multiplying $(22)_2$ by J and integrating it with respect to x over $[0,1]$, we can obtain (34). Since this can be done as exactly showed in [21], we omit the details.

Similarly, multiplying $(22)_4$ by H and integrating it over $[0,1]$, we then prove (35).

From the equation of $(22)_2$, we have

$$J_t^2 = \Big\{ -\Big(\frac{(\bar{j}+J)^2}{N+\bar{n}} - \frac{\bar{j}^2}{\bar{n}} + p(N+\bar{n}) - p(\bar{n})\Big)_x + (N+\bar{n})\Phi_x + \bar{\phi}_x N - J\Big\}^2.$$

Integrating it over $[0,1]$ and applying (34), gives (36). In a way similar to $(22)_4$ plus the estimate (36) then yields (37). $\square$

Lemma 3.4 (Key energy estimates)**.** *Under the hypotheses of Theorem 3.1, it holds*

$$\begin{aligned}
&\|(N, N_x, N_t, H, H_x, H_t, \Phi_{xx})(t)\|^2 \\
&+ \int_0^t e^{-\mu(t-s)}\|(N, N_x, N_t, H, H_x, H_t, \Phi_{xx})(s)\|^2 ds \\
&\le Ce^{-2\mu t}[\|(N_0, H_0)\|_1^2 + \|(J_0, K_0)\|_1^2]
\end{aligned} \tag{38}$$

provided with $M(T) + \delta \ll 1$.

Proof. By a heuristic observation to (26) and (27), we ingeniously take the following procedure:

$$(26)\cdot \bar{h}(N + 2N_t) + (27)\cdot \bar{n}(H + 2H_t), \tag{39}$$

that is,

$$\begin{aligned}
&\Big\{\bar{h}\Big(NN_t + \frac{1}{2}N^2 + N_t^2\Big)\Big\}_t \\
&\quad + \Big\{\bar{n}\Big(HH_t + \frac{1}{2}H^2 + H_t^2\Big)\Big\}_t \\
&\quad + \bar{h}N_t^2 + \bar{n}H_t^2 + \bar{n}\bar{h}\Phi_{xx}^2 + \Big\{\bar{n}\bar{h}\Phi_{xx}^2\Big\}_t \\
&\quad - \Big\{F_{1x}\cdot \bar{h}(N + 2N_t)\Big\}_x - \Big\{G_{1x}\cdot \bar{n}(H + 2H_t)\Big\}_x \\
&\quad + F_{1x}\bar{h}(N_x + 2N_{xt}) + F_{1x}\bar{h}_x(N + 2N_t) \\
&\quad + G_{1x}\bar{n}(H_x + 2H_{xt}) + G_{1x}\bar{n}_x(H + 2H_t) \\
&= F_2\bar{h}(N + 2N_t) + G_2\bar{n}(H + 2H_t).
\end{aligned} \tag{40}$$

Integrating (40) over $[0,1]$, we have

$$\begin{aligned}
&\frac{d}{dt}\int_0^1 \bar{h}\Big(NN_t+\frac{1}{2}N^2+N_t^2\Big)dx\\
&\quad+\frac{d}{dt}\int_0^1 \bar{n}\Big(HH_t+\frac{1}{2}H^2+H_t^2\Big)dx\\
&\quad+\int_0^1[\bar{h}N_t^2+\bar{n}H_t^2+\bar{n}\bar{h}\Phi_{xx}^2]dx+\frac{d}{dt}\int_0^1\bar{n}\bar{h}\Phi_{xx}^2dx\\
&\quad+\int_0^1 F_{1x}\bar{h}(N_x+2N_{xt})dx+\int_0^1 F_{1x}\bar{h}_x(N+2N_t)dx\\
&\quad+\int_0^1 G_{1x}\bar{n}(H_x+2H_{xt})dx+\int_0^1 G_{1x}\bar{n}_x(H+2H_t)dx\\
&=\int_0^1 F_2\bar{h}(N+2N_t)dx+\int_0^1 G_2\bar{n}(H+2H_t)dx.
\end{aligned}\tag{41}$$

Applying (34) and (36), by a straightforward but tedious computation, we have

$$\begin{aligned}
&\int_0^1 F_{1x}\bar{h}(N_x+2N_{xt})dx\\
&=\int_0^1\Big(\frac{(\bar{j}+J)^2}{N+\bar{n}}-\frac{\bar{j}^2}{\bar{n}}+p(N+\bar{n})-p(\bar{n})\Big)_x\bar{h}(N_x+2N_{xt})dx\\
&\geq\int_0^1\bar{h}\Big(p'(\bar{n})-\frac{\bar{j}^2}{\bar{n}^2}\Big)N_x^2dx+\frac{d}{dt}\int_0^1\bar{h}\Big(p'(\bar{n})-\frac{\bar{j}^2}{\bar{n}^2}\Big)N_x^2dx\\
&\quad+\frac{d}{dt}\int_0^1\bar{h}(N_x^2+N_xJ_{0x})\Big(p'(N+\bar{n})-p'(\bar{n})-\frac{(\bar{j}+J)^2}{(N+\bar{n})^2}+\frac{\bar{j}^2}{\bar{n}^2}\Big)dx\\
&\quad-O(1)[\delta+M(T)]\|(N,N_x,N_t,J,J_t)(t)\|^2\\
&\geq\int_0^1\bar{h}\Big(p'(\bar{n})-\frac{\bar{j}^2}{\bar{n}^2}\Big)N_x^2dx+\frac{d}{dt}\int_0^1\bar{h}\Big(p'(\bar{n})-\frac{\bar{j}^2}{\bar{n}^2}\Big)N_x^2dx\\
&\quad+\frac{d}{dt}\int_0^1\bar{h}(N_x^2+N_xJ_{0x})\Big(p'(N+\bar{n})-p'(\bar{n})-\frac{(\bar{j}+J)^2}{(N+\bar{n})^2}+\frac{\bar{j}^2}{\bar{n}^2}\Big)dx\\
&\quad-O(1)[\delta+M(T)]\|(N,N_x,N_t,H,H_x,H_t)(t)\|^2\\
&\quad-O(1)e^{-2\mu_0t}\|(J_0,H_0)\|^2.
\end{aligned}\tag{42}$$

Similarly, we have

$$\begin{aligned}
&\int_0^1 G_{1x}\bar{n}(H_x+2H_{xt})dx\\
&\geq\int_0^1\bar{n}\Big(q'(\bar{h})-\frac{\bar{k}^2}{\bar{h}^2}\Big)H_x^2dx+\frac{d}{dt}\int_0^1\bar{n}\Big(q'(\bar{h})-\frac{\bar{k}^2}{\bar{h}^2}\Big)H_x^2dx\\
&\quad+\frac{d}{dt}\int_0^1\bar{n}(H_x^2+H_xK_{0x})\Big(q'(H+\bar{h})-q'(\bar{h})-\frac{(\bar{k}+K)^2}{(H+\bar{h})^2}+\frac{\bar{k}^2}{\bar{h}^2}\Big)dx\\
&\quad-O(1)[\delta+M(T)]\|(N,N_x,N_t,H,H_x,H_t)(t)\|^2\\
&\quad-O(1)e^{-2\mu_0t}\|(J_0,H_0)\|^2.
\end{aligned}\tag{43}$$

On the other hand, from (28)-(31) and (33), it is easy to see

$$\Big|\int_0^1 F_{1x}\bar{h}_x(N+2N_t)dx\Big| \le C[\delta+M(T)]\|(N,N_x,N_t)(t)\|^2, \tag{44}$$

$$\Big|\int_0^1 G_{1x}\bar{n}_x(H+2H_t)dx\Big| \le C[\delta+M(T)]\|(H,H_x,H_t)(t)\|^2, \tag{45}$$

$$\Big|\int_0^1 F_2\bar{h}_x(N+2N_t)dx\Big| \le C[\delta+M(T)]\|(N,N_x,H)(t)\|^2, \tag{46}$$

$$\Big|\int_0^1 G_2\bar{n}_x(H+2H_t)dx\Big| \le C[\delta+M(T)]\|(N,H,H_x)(t)\|^2. \tag{47}$$

Substituting (42)-(47) to (41), we have

$$\begin{aligned}
&\frac{d}{dt}\int_0^1 E_1(N,N_x,N_t,H,H_x,H_t,\Phi_{xx})dx\\
&\quad+\frac{d}{dt}\int_0^1 E_2(N,N_x,J,H,H_x,K)dx\\
&\quad+\int_0^1 E_3(N,N_x,N_t,H,H_x,H_t,\Phi_{xx})dx\\
&\le Ce^{-2\mu_0 t}\|(N_0,N_{0x},J_0,H_0,H_{0x},K_0\|^2\\
&\quad+C[\delta+M(T)]\|(N,N_x,N_t,H,H_x,H_t)(t)\|^2,
\end{aligned} \tag{48}$$

where

$$\begin{aligned}
&E_1(N,N_x,N_t,H,H_x,H_t,\Phi_{xx})\\
&=\bar{h}\Big(NN_t+\frac{1}{2}N^2+N_t^2\Big)+\bar{h}\Big(p'(\bar{n})-\frac{\bar{j}^2}{\bar{n}^2}\Big)N_x^2\\
&\quad+\bar{n}\Big(HH_t+\frac{1}{2}H^2+H_t^2\Big)+\bar{n}\Big(q'(\bar{h})-\frac{\bar{k}^2}{\bar{h}^2}\Big)H_x^2+\bar{n}\bar{h}\Phi_{xx}^2,
\end{aligned} \tag{49}$$

and

$$\begin{aligned}
&E_2(N,N_x,J,H,H_x,K)\\
&=\bar{h}(N_x^2+N_xJ_{0x})\Big(p'(N+\bar{n})-p'(\bar{n})-\frac{(\bar{j}+J)^2}{(N+\bar{n})^2}+\frac{\bar{j}^2}{\bar{n}^2}\Big)\\
&\quad+\bar{n}(H_x^2+H_xK_{0x})\Big(q'(H+\bar{h})-q'(\bar{h})-\frac{(\bar{k}+K)^2}{(H+\bar{h})^2}+\frac{\bar{k}^2}{\bar{h}^2}\Big),
\end{aligned} \tag{50}$$

and

$$\begin{aligned}
&E_3(N,N_x,N_t,H,H_x,H_t,\Phi_{xx})\\
&=\bar{h}N_t^2+\bar{n}H_t^2+\bar{n}\bar{h}\Phi_{xx}^2+\bar{h}\Big(p'(\bar{n})-\frac{\bar{j}^2}{\bar{n}^2}\Big)N_x^2+\bar{n}\Big(q'(\bar{h})-\frac{\bar{k}^2}{\bar{h}^2}\Big)H_x^2.
\end{aligned} \tag{51}$$

Notice that,

$$\bar{C}_1(N^2+N_t^2)\le NN_t+\frac{1}{2}N^2+N_t^2\le\bar{C}_2(N^2+N_t^2), \tag{52}$$

and

$$\bar{C}_1(H^2+H_t^2)\le HH_t+\frac{1}{2}H^2+H_t^2\le\bar{C}_2(H^2+H_t^2) \tag{53}$$

for some positive constants $\bar{C}_1$ and $\bar{C}_2$. From (52), (53), (12) and (13), we then obtain

$$\begin{aligned}
&C_1(N^2+N_x^2+N_t^2+H^2+H_x^2+H_t^2+\Phi_{xx})\\
&\le E_1(N,N_x,N_t,H,H_x,H_t,\Phi_{xx})\\
&\le C_2(N^2+N_x^2+N_t^2+H^2+H_x^2+H_t^2+\Phi_{xx}),
\end{aligned} \tag{54}$$

and

$$\begin{aligned}
&C_3(N^2+N_x^2+N_t^2+H^2+H_x^2+H_t^2+\Phi_{xx})\\
&\le E_3(N,N_x,N_t,H,H_x,H_t,\Phi_{xx})\\
&\le C_4(N^2+N_x^2+N_t^2+H^2+H_x^2+H_t^2+\Phi_{xx}),
\end{aligned} \tag{55}$$

and

$$\begin{aligned}
&|E_2(N,N_x,J,H,H_x,K)|\\
&\le C[\delta+M(T)]\|(N,N_x,J,H,H_x,K)(t)\|^2\\
&\le C[\delta+M(T)]\Big(e^{-2\mu_0 t}\|(J_0,H_0)\|^2+\|(N,N_x,N_t,H,H_x,H_t)(t)\|^2\Big),
\end{aligned} \tag{56}$$

for some positive constants C_1, C_2, C_3 and C_4.

Multiplying (48) by $e^{2\mu t}$ for a positive constant μ which will be determined later, and integrating the resultant equation over $[0,t]$, we get

$$\begin{aligned}
&e^{2\mu t}\int_0^1 E_1(N,N_x,N_t,H,H_x,H_t,\Phi_{xx})dx\\
&\quad+e^{2\mu t}\int_0^1 E_2(N,N_x,J,H,H_x,K)dx\\
&\quad+\int_0^t e^{2\mu s}\int_0^1 E_3(N,N_x,N_t,H,H_x,H_t,\Phi_{xx})dx\\
&\quad-2\mu\int_0^t e^{2\mu s}\int_0^1 E_1(N,N_x,N_t,H,H_x,H_t,\Phi_{xx})dx\\
&\quad-2\mu\int_0^t e^{2\mu s}\int_0^1 E_2(N,N_x,J,H,H_x,K)dx\\
&\le\int_0^1 E_1(N,N_x,N_t,H,H_x,H_t,\Phi_{xx})|_{t=0}dx\\
&\quad+\int_0^1 E_2(N,N_x,J,H,H_x,K)|_{t=0}dx\\
&\quad+\frac{C}{2(\mu_0-\mu)}(1-e^{-2(\mu_0-\mu)t})\|(N_0,N_{0x},J_0,H_0,H_{0x},K_0\|^2\\
&\quad+C[\delta+M(T)]\int_0^t e^{2\mu s}\|(N,N_x,N_t,H,H_x,H_t)(s)\|^2ds.
\end{aligned} \tag{57}$$

Applying (54)-(56) to (57), we have

$$
\begin{aligned}
&C_1 e^{2\mu t}\|(N, N_x, N_t, H, H_x, H_t, \Phi_{xx})(t)\|^2 \\
&\quad -Ce^{2\mu t}[\delta + M(T)]\|(N, N_x, N_t, H, H_x, H_t)(t)\|^2 \\
&\quad +C_3 \int_0^t e^{2\mu s}\|(N, N_x, N_t, H, H_x, H_t, \Phi_{xx})(s)\|^2 ds \\
&\quad -2\mu C_2 \int_0^t e^{2\mu s}\|(N, N_x, N_t, H, H_x, H_t, \Phi_{xx})(s)\|^2 ds \\
&\quad -2\mu C[\delta + M(T)] \int_0^t e^{2\mu s}\|(N, N_x, N_t, H, H_x, H_t)(s)\|^2 ds \\
&\le C\|(N, N_x, N_t, H, H_x, H_t, \Phi_{xx})(0)\|^2 \\
&\quad +C[\delta + M(T)]\Big(\|(J_0, H_0)\|^2 + \|(N, N_x, N_t, H, H_x, H_t)(0)\|^2\Big) \\
&\quad +\frac{C}{2\mu_0 - 2\mu}(1 - e^{-2(\mu_0-\mu)t})\|(N_0, N_{0x}, J_0, H_0, H_{0x}, K_0\|^2 \\
&\quad +C[\delta + M(T)] \int_0^t e^{2\mu s}\|(N, N_x, N_t, H, H_x, H_t)(s)\|^2 ds. \qquad (58)
\end{aligned}
$$

Let

$$
0 < \mu < \min\Big\{\mu_0, \frac{C_3}{2C_2}\Big\}, \qquad (59)
$$

and

$$
\delta + M(T) \ll 1,
$$

then (58) yields the desired estimate

$$
\begin{aligned}
&\|(N, N_x, N_t, H, H_x, H_t, \Phi_{xx})(t)\|^2 \\
&+ \int_0^t e^{-2\mu(t-s)}\|(N, N_x, N_t, H, H_x, H_t, \Phi_{xx})(t)\|^2 ds \\
&\le Ce^{-2\mu t}[\|(N_0, H_0)\|_1^2 + \|(J_0, K_0)\|_1^2].
\end{aligned}
$$

The proof is complete. $\square$

Lemma 3.5. *It holds*

$$
\begin{aligned}
&\|(N_{xx}, N_{xt}, H_{xx}, H_{xt}, \Phi_{xxx})(t)\|^2 \\
&+ \int_0^t e^{-2\mu(t-s)}\|(N_{xx}, N_{xt}, H_{xx}, H_{xt}, \Phi_{xxx})(s)\|^2 ds \\
&\le Ce^{-2\mu t}[\|(N_0, H_0)\|_2^2 + \|(J_0, K_0)\|_2^2] \qquad (60)
\end{aligned}
$$

provided that $M(T) + \delta \ll 1$.

Proof. By taking

$$
\int_0^t e^{2\mu s} \int_0^1 \Big\{\partial_x(26) \cdot \bar{h}(N_x + 2N_{xt}) + \partial_x(27) \cdot \bar{n}(H_x + 2H_{xt})\Big\} dx ds,
$$

as showed in Lemma 3.4, we can similarly prove (60). The detail is omitted. $\square$

Combining Lemma 3.2-Lemma 3.5, we immediately obtain the following estimate.

Lemma 3.6. *It holds*

$$\begin{aligned}&\|(N,J,H,K)(t)\|_2+\|(N_t,J_t,H_t,K_t)(t)\|_1+\|\Phi(t)\|_3\\&+\int_0^t e^{-\mu(t-s)}[\|(N,H)(s)\|_2^2+\|(N_t,H_t,\Phi_{xx})(s)\|_1^2]ds\\&\le Ce^{-\mu t}.\end{aligned} \tag{61}$$

provided that $\delta+M(T)\ll 1$*.*

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REFERENCES

[1] G. Ali, *Global existence of smooth solutions of the N-dimensional Euler-Possion model*, SIAM J. Math. Anal., **35** (2003), 389–422.

[2] G. Ali, D. Bini and D. Rionero, *Global existence and relaxation limit for smooth solutions to the Euler-Possion model for semiconductors*, SIAM J. Math. Anal., **32** (2000), 572–587.

[3] K. Bløtekjær, *Transport equations for electrons in two-valley semiconductors*, IEEE Trans. Electron Devices, **17** (1970), 38–47.

[4] G. Chen, J. Jerome and B. Zhang, *Particle hydrodynamic moment models in biology and microelectronics: Singular relaxation limits*, Nonlinear Anal., **30** (1997), 233–244.

[5] G. Chen and D. Wang, *Convergence of shock capturing schemes for the compressible Euler-Poisson equations*, Comm. Math. Phys., **179** (1996), 333–364.

[6] P. Degond and P. A. Markowich, *On a one-dimensional steady-state hydrodynamic model*, Appl. Math. Lett., **3** (1990), 25–29.

[7] W. Fang and K. Ito, *Steady-state solutions of a one-dimensional hydrodynamic model for semiconductors*, J. Differential Equations, **133** (1997), 224–244.

[8] I. Gamba, *Stationary transonic solutions of a one-dimensional hydrodynamic model for semiconductor*, Comm. Partial Diff. Eqns, **17** (1992), 553–577.

[9] I. Gasser, L. Hsiao and H.-L. Li, *Large time behavior of solutions of the bipolar hydrodynamical model for semiconductors*, J. Differential Equations, **192** (2003), 326–359.

[10] I. Gasser and R. Natalini, *The energy transport and the drift diffusion equations as relaxation limits of the hydrodynamic model for semiconductors*, Quart. Appl. Math., **57** (1996), 269–282.

[11] Y. Guo and W. Strauss, *Stability of semiconductor states with insulating and contact boundary conditions*, Arch. Rational Mech. Anal., **179** (2005), 1–30.

[12] L. Hsiao and K. Zhang, *The global weak solution and relaxation limits of the initial boundary value problem to the bipolar hydrodynamic model for semiconductors*, Math. Models Methods Appl. Sci., **10** (2000), 1333–1361.

[13] L. Hsiao and K. Zhang, *The relaxation of the hydrodynamic model for semiconductors to drift diffusion equations*, J. Differential Equations, **165** (2000), 315–354.

[14] F.-M. Huang and Y.-P. Li, *Large time behavior and quasineutral limit of solutions to a bipolar hydrodynamic model with large data and vacuum*, Discrete Contin. Dyn. Syst., **24** (2009), 455–470.

[15] F.-M. Huang, M. Mei and Y. Wang, *Large time behavior of solutions to n-dimensional bipolar hydrodynamic model for semiconductors*, SIAM J. Math. Anal., **43** (2011), 1595–1630.

[16] F.-M. Huang, M. Mei, Y. Wang and T. Yang, *Long-time behavior of solutions for bipolar hydrodynamic model of semiconductors with boundary effects,* SIAM J. Math. Anal., **44**, (2012), 1134–1164.

[17] F.-M. Huang, M. Mei, Y. Wang and H. Yu, *Asymptotic convergence to stationary waves for unipolar hydrodynami model of semiconductors*, SIAM J. Math. Anal., **43** (2011), 411–429.

[18] F.-M. Huang, M. Mei, Y. Wang and H. Yu, *Asymptotic convergence to planar stationary waves for multi-dimensional unipolar hydrodynamic model of semiconductors*, J. Differential Equations, **251** (2011), 1305–1331.

[19] J. W. Jerome, *Steady Euler-Poisson system: a differential/integral equation formulation with general constitutive relations*, Nonlinear Anal., **71** (2009), e2188–e2193.

[20] A. Jüngel, "Quasi-hydrodynamic Semiconductor Equations," Progress in Nonlinear Differential Equations and their Applications, Vol 41, Birkhäuser Verlag, Besel-Boston-Berlin, 2001.

[21] H.-L. Li, P. Markowich and M. Mei, *Asymptotic behavior of solutions of the hydrodynamic model of semiconductors*, Proc. Royal Soc. Edinburgh, Sect. A, **132** (2002), 359–378.

[22] H.-L. Li, P. Markowich and M. Mei, *Asymptotic behavior of subsonic entropy solutions of the isentropic Euler-Poisson equations*, Quart. Appl. Math., **60** (2002), 773–796.

[23] C.-K. Lin, C.-T. Lin and M. Mei, *Asymptotic behavior of solution to nonlinear damped p-system with boundary effect,* Int. J. Numer. Anal. Model. Ser. B, **1** (2010), 70–92.

[24] T. Luo, R. Natalini and Z. Xin, *Large time behavior of the solutions to a hydrodynamic model for semiconductors*, SIAM J. Appl. Math., **59** (1998), 810–830.

[25] P. Marcati, M. Mei and B. Rubino, *Optimal convergence rates to diffusion waves for solutions of the hyperbolic conservation laws with damping*, J. Math. Fluid Mech., **7** (2005), suppl. 2, S224–S240

[26] P. Marcati and R. Natalini, *Weak solutions to a hydrodynamic model for semiconductors and relaxation to the drift-diffusion equation*, Arch. Rational Mech. Anal., **129** (1995), 129–145.

[27] P. A. Markowich, C. A. Ringhofer and C. Schmeiser, "Semiconductor Equations," Springer-Verlag, Vienna, 1990.

[28] M. Mei, *Best asymptotic profile for hyperbolic p-sytem with damping*, SIAM J. Math. Anal., **42** (2010), 1–23.

[29] M. Mei and Y. Wang, *Stability of stationary waves for full Euler-Poisson system in multi-dimensional space*, Commun. Pure Appl. Anal., **11** (2012), 1775–1807.

[30] R. Natalini, *The bipolar hydrodynamic model for semiconductors and the drift-diffusion equations*, J. Math. Anal. Appl., **198** (1996), 262–281.

[31] S. Nishibata and M. Suzuki, *Asymptotic stability of a stationary solution to a hydrodynamic model of semiconductors*, Osaka J. Math., **44** (2007), 639–665.

[32] S. Nishibata and M. Suzuki, *Asymptotic stability of a stationary solution to a thermal hydrodynamic model for semiconductors*, Arch. Rational Mech. Anal., **192** (2009), 187–215.

[33] F. Poupaud, M. Rascle and J.-P. Vila, *Global solutions to the isothermal Euler-Poisson system with arbitrarily large data*, J. Differential Equations, **123** (1995), 93–121.

[34] A. Sitenko and V. Malnev, "Plasma Physics Theory," Applied Mathematics and Mathematical Computation, 10. Chapman & Hall, London, 1995.

[35] N. Tsuge, *Existence and uniqueness of stationary solutions to one-dimensional bipolar hydrodynamic model of semiconductors*, Nonlinear Analysis, **73** (2010), 779–787.

[36] B. Zhang, *Convergence of the Godunov scheme for a simplified one-dimensional hydrodynamic model for semiconductor devices*, Comm. Math. Phys., **157** (1993), 1–22.

[37] C. Zhu and H. Hattori, *Stability of steady state solutions for an isentropic hydrodynamic model of semiconductors of two species*, J. Differential Equations, **166** (2000), 1–32.

[38] C. Zhu and H. Hattori, *Asymptotic behavior of the solution to a nonisentropic hydrodynamic model of semiconductors,* J. Differential Equations, **144** (1998), 353–389.

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